Magnetic Navigation of Artificial Bacteria Flagella in Blood and Water

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**Artificial Microswimmers**


**Cargo Transport**


**Micro-manipulation**


**Assisted Fertilization**


**Targeted Drug Delivery**

Outline

1. Independent Control and Path Planning of artificial bacterial flagella

2. Artificial bacterial flagella swimming in blood
Artificial Bacterial Flagella (ABF)

Re \approx 10^{-4} \ll 1 \Rightarrow \text{Stokes flow.}

\text{Magnetic torque: } T = m \times B

\begin{align*}
\dot{q} &= \frac{1}{2} q \otimes \hat{\Omega}, \\
\dot{x} &= V, \\
V^B &= \mathcal{B} T^B, \\
\Omega^B &= \mathcal{C} T^B.
\end{align*}

ODE model

\[
\begin{bmatrix}
V^B \\
\Omega^B
\end{bmatrix} =
\begin{bmatrix}
A & B \\
B & C
\end{bmatrix}
\begin{bmatrix}
F^B \\
T^B
\end{bmatrix}
\]
Different geometries allow independent control

The geometry governs the response to the rotation frequency of the magnetic field.
Swimming different distances along a direction

Constraint: All swimmers are subjected to the same magnetic field

Velocity matrix: $U_{ij} = v_i(\omega_{c,j})$

(signed) distance made by swimmer $i$: $d_i = \sum_j U_{ij} t_j s_j = U_{ij} b_j$

where $s_j = -1$ (clockwise) or $s_j = +1$ (anti-clockwise)

$\Rightarrow b = U^{-1}d$
Gathering swimmers to a target

In 3 dimensions: 3 steps to reach the target

1. Gather on a plane
2. Gather on a line
3. Gather at the target
Independent control in free space
Reinforcement Learning

State
- positions $x - x_G$
- orientations $q$

Policy

Reward
$$r_t = \sum_{i=1}^{n_b} \left( \| x_i(t-1) - x^G_i \|_2^2 - \| x_i(t) - x^G_i \|_2^2 - K \Delta t \right)$$
Terminal reward has bonus $K_f$
if $\| x_i(T) - x^G_i \|_2 \leq L_{\text{max}}$

Action
- Magnetic field frequency of rotation $\omega$
- Magnetic field orientation

Environment

Initial conditions
Independent control in free space: RL

![Graphs showing analytic and RL results](image-url)
Independent control with a background flow

Taylor-Green stationary flow

\[ \mathbf{u}_\infty(\mathbf{r}) = \begin{bmatrix} A \cos ax \sin by \sin cz \\ B \sin ax \cos by \sin cz \\ C \sin ax \sin by \cos cz \end{bmatrix} \]
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Blood Model

Bending Energy

\[ E_b = 2\kappa_b \oint (H - H_0)^2 \, dA \]


Area and Volume penalization

\[ E_A = k_A \frac{(A - A_0)^2}{A_0}, \quad E_V = k_V \frac{(V - V_0)^2}{V_0} \]


Dissipation forces

\[ \mathbf{f}_i^{\text{visc}} = - \sum_j \gamma \left( \mathbf{v}_{ij} \cdot \mathbf{e}_{ij} \right) \mathbf{e}_{ij} \]


Shear Energy

\[ E_s = \frac{K_a}{2} \oint (a^2 + a_3a^3 + a_4a^4) \, dA_0 + \mu \oint (\beta + b_1a\beta + b_2\beta^2) \, dA_0 \]

with respect to stress-free shape:

**Solvent Model**

**Newton motion**

\[ \ddot{r}_i = v_i, \]

\[ \dot{v}_i = \frac{1}{m_i} f_i, \]

**Bounce Back on the membrane**

\[ \mathbf{v}(t + dt) = 2 \mathbf{v}_{RBC}(t_{collision}) - \mathbf{v}(t) \]

**Dissipative Particle Dynamics interactions**

\[ f^C_{ij} = a \nu \left( r_{ij} \right) \mathbf{e}_{ij}, \]

\[ f^D_{ij} = -\gamma \nu_D \left( r_{ij} \right) \left( \mathbf{e}_{ij} \cdot \mathbf{v}_{ij} \right) \mathbf{e}_{ij}, \]

\[ f^R_{ij} = \sigma_{ij} \nu_R \left( r_{ij} \right) \mathbf{e}_{ij}. \]

- **hydrostatic pressure**
- **viscosity**
- **fluctuations**
ABFs swimming in blood

Ht = 10\%

Ht = 20\%

ABFs swim faster at higher hematocrit
Summary

• Reinforcement learning is a good tool for independent control of multiple swimmers under a uniform magnetic field.

• Artificial bacterial flagella can navigate in blood efficiently if the magnetic torque is high enough.

• Opens the road to optimize the design of single and swarms of swimmers.