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Set 4

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In this exercise, you will learn about fitting data using Lagrange interpolation. The first exercise is writing down the pseudocode of Lagrange Interpolation. The second exercise is a numerical application of Lagrange interpolation. The third exercise compares Lagrange interpolation to Least Squares fitting. In **Notebook 4.1** you will write a code to perform Lagrange interpolation on equally spaced points. In doing so, you will observe Runge's phenomenon, namely interpolation divergence, that can occur near the endpoints when using this technique.

Question 1: Lagrange interpolation pseudocode

Write a pseudocode for Lagrange interpolation. Your function should take as inputs two arrays $\mathbf{x}$ and $\mathbf{y}$ of size N, as well as a position $\bar{x}$. It should output the interpolated value $\bar{y}$ at $\bar{x}$ using the input data.

Implement this algorithm in Notebook 4.1.

Question 2: Lagrange Interpolation

You sampled the function $f(x) = -\frac{2}{3}x^3 + \frac{5}{3}x^2 + 4x$ at the points $\{x_i\} = \{0, 1, 3\}$ and wish to compute the Lagrange Interpolating polynomial for this dataset.

a) Compute all the Lagrange basis functions for the above data.

b) Construct the Lagrange interpolation of the above data.

c) Interpolate the above data at $x = 2$ using your previous results and compute the interpolation error. How would your interpolation error change if you sampled a fourth point?

Question 3: LSQ vs Lagrange

As part of an experimental campaign you have gathered measurements of the distance a car moved with respect to time:

a) Based on the measurements you want to predict the position of the car a time $t = 5$ sec. What possible methods could you use to make this prediction? Is there any assumptions you can make on the relation between position and time? What would be the effect of noise in the measurements? Justify which method you would use depending on each assumption.

t [s]	x(t) [m]
1	12.5
2	30
3	52.5
4	80

b) Thinking back about your Physics I course, you make the assumption that the car is under constant acceleration a and started with an initial velocity v_0. The time-dependent position of the car can thus be described by the following relation:

$$x(t) = v_0 \cdot t + \frac{1}{2}\alpha \cdot t^2$$

You first decide to solve this problem using a least squares fit. Taking into consideration your new assumption, which basis functions would you use? Find the values α and v_0 and $x[5]$ by performing a LSQ fit.

c) Based on your previous results, what can you tell about the noise present in the measurements? Would there have been a faster way to find α and v_0?

d) Using the same data and the same assumption regarding the behavior of the position trajectory, perform an interpolation using Lagrange polynomials. Do you get the same value for α and v_0?

e) What would happen if you didn't know that the data followed a quadratic relation and used all the available data to perform a Lagrange interpolation?

f) Assume now that you get new data but due to wear, the speedometer of the moving car is malfunctioning and produces an error in velocity measurement that follows the normal random distribution with mean 0 and standard deviation σ. What would be the effect of noise on the resulting interpolation function with each of the methods tested above? Which method would be better suited in this case?