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Set 11 - 2D Diffusion, ADI, Thomas algorithm

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Hand in (optional): **December 17**, 2018 23:59

Question 1: Diffusion in 2D using ADI scheme

Heat flow in a medium can be described by the diffusion equation of the form

$$\frac{\partial \rho(x,y,t)}{\partial t} = D\nabla^2 \rho(x,y,t) \tag{1}$$

where $\rho(x,y,t)$ is a measure for the amount of heat at position $\boldsymbol{r}$ and time t and the diffusion coefficient D is constant. Lets define the domain Ω in two dimensions as $x, y \in [-1,1]$. We will use the boundary condition:

$$\rho(x,y,t) = 0 \quad \forall\, t \geq 0 \,\text{and}\, (x,y) \notin \Omega \tag{2}$$

and an initial distribution:

$$\rho(x,y,0) = \begin{cases} 1 & |x,y| < 1/2 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

a) Discretize equation (1) using the Alternating Direction Implicit (ADI) scheme. Write down the resulting system in matrix form. What do you observe? Comment on your choice of method/algorithm for the solution of the resulting implicit scheme and explain why this choice is justified.

b) Use the provided skeleton code (`diffusion2d_adi_openmp.cpp`) to solve the 2D diffusion problem on a uniform grid. Implement the missing code parts in all sections marked by `TODO`. Use Thomas algorithm for the solution of the implicit systems resulting from the ADI scheme.

c) Parallelize your code using OpenMP. Comment on any complexity that would arise if you chose to parallelize the ADI scheme with MPI.

d) Compute an approximation to the integral of ρ over the entire domain in `compute_diagnostics` and plot the result as a function of time using the parameters D =1, $L = 2$ and $N = 256$.