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Set 12 - Particle Strength Exchange (PSE) with MPI

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Question 1: PSE for 2D diffusion

We want to use the Particle Strength Exchange method to simulate the diffusion equation

$$\frac{\partial u(\mathbf{x},t)}{\partial t} = D\Delta u(\mathbf{x},t), \tag{1}$$

in a two dimensional domain $[0,1]^2$ with periodic boundary conditions and $D = 1$ and a mesh of $N \times N$ particles.

Complete the skeleton code provided in `skeleton_code/PSE_MPI.cpp` such that the program uses MPI to simulate the diffusion with 4 processes, where each process works on a quarter of the domain and uses ghost values. You may follow the `TODO` comments that guide you through the exercise. You can add member variables and methods where needed.

Use the following mollification kernel:

$$\eta(\mathbf{x}) = \frac{16}{\pi^2}\frac{1}{|\mathbf{x}|^8+1}, \tag{2}$$

with a support of $\epsilon = 10h$, where h is the grid spacing. The PSE discretization of Equation (1) reads as

$$u_p^{n+1} = u_p^n + \frac{D\delta t}{\epsilon^2}\sum_q (u_q^n - u_p^n)V_p\eta_\epsilon(\mathbf{x}_q - \mathbf{x}_p), \tag{3}$$

where V_p is the particle volume. u_p^n is the quantity u carried by particle p at time $t^n = n\delta t$. The forward Euler scheme has been used to integrate in time. The index q denotes the particles that are within the support of the particle with index p. Note that $\eta_\epsilon(\mathbf{x}) = \epsilon^{-2}\eta(\mathbf{x}/\epsilon)$.

Set the number of particles $N \times N$ to 80×80 and the timestep as

$$\delta t = \frac{h^2}{2D}, \tag{4}$$

with the uniform grid spacing given by $h = 1/N$.

As initial conditions, set

$$u(x,y,0) = \sin(2\pi x)\sin(2\pi y), \tag{5}$$

such that the analytical solution is

$$u(x,y,t) = \sin(2\pi x)\sin(2\pi y)\exp(-8D\pi^2 t). \tag{6}$$