

$$e) b*=(++a)+b \rightarrow b*=(6+2) \rightarrow b*=8 \rightarrow 16.$$

$$f) a-(a*(+(-b))) \rightarrow 5-(5*(+(-2))) \rightarrow 5-(5*(+-2)) \rightarrow 5-(5*-2) \rightarrow 5--10 \rightarrow 15.$$

$$i) (b++)+(-a) \rightarrow 2+4 \rightarrow 6.$$

**Solution to Exercise 16.** For any positive  $d$ , we can uniquely write  $n$  in the form

$$n = xd + y,$$

where  $x, y \in \mathbb{N}$  and  $y < d$ . In fact, these define `div` and `mod` via  $x = n \text{ div } d$  and  $y = n \text{ mod } d$ .

For  $n = a$ ,  $d = bc$ , we get

$$a = pbc + q,$$

where  $p, q \in \mathbb{N}$  and  $q < bc$ . In particular,  $p = a \text{ div } (bc)$ . It remains to prove that

$$(a \text{ div } b) \text{ div } c = p.$$

To this end, we use  $n = q$ ,  $d = b$  to get

$$q = rb + s,$$

where  $r, s \in \mathbb{N}$ ,  $s < b$  and  $r < c$  (since  $rb \leq q < cb$ ). Then we have

$$a = (pc + r)b + s, s < b,$$

and this means that

$$a \text{ div } b = pc + r, r < c.$$

This in turn means that  $p = (a \text{ div } b) \text{ div } c$ .

The implication holds as long as the mathematical value of the expression  $b*c$  is representable as an `unsigned int` value. This holds since the other operations involving `/` are always error-free. If there is an overflow in  $b*c$ , the expressions  $a/b/c$  and  $a/(b*c)$  might yield different values.