Eidgenössische Technische Hochschule Zürich — *Ecole polytechnique fédérale de Zurich* / *Politecnico federale di Zurigo* / *Swiss Federal Institute of Technology Zurich*

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Advanced Computational Science
SS 2009

Serie 11 — Ausgabe: May 12, 2009 — Abgabe: May 19, 2009

Guidelines

▷ Submission: Submit a written or typed report with your answers and graphics.

▷ Code: Send your source code (and makefile) to the TA of your session. Use "[ACS] Homework11" in the subject of your email.

▷ Credit: Your report should list all the contributors.

▷ Bugs: Print-outs of your source code are *not* required in your report, unless you have bugs. Help us give you partial marks.

▷ Deadline: Assignments have to be handed in at the beginning of the next exercise session.

Exercise 1 (40 points) Biological Patterns

During the early stages of development of animals, morphogens, such as the epidermal growth factor, govern the pattern of tissue development and, in particular, the positions of specialized cell types within the tissue. This mechanism is responsible for the stripes of zebras, the spots of leopards, etc...

The phenomenon of emergence of complex patterns formed by morphogens, can be described by reaction-diffusion systems. Here we use the Gray-Scott differential equations to model the evolution of two morphogenic species in 2 dimensions

$$\frac{\partial u}{\partial t} = D_u \Delta u - uv^2 + F(1-u), \qquad (1)$$

$$\frac{\partial v}{\partial t} = D_v \Delta v + uv^2 - (F+\kappa)v \qquad (2)$$

where $u(\vec{x},t), v(\vec{x},t)$ denote the space and time dependent concentrations of the species U and V.

Part 1 (20 points)

Implement a numerical algorithm to solve the Gray-Scott differential equations using the simple explicit euler scheme and second-order finite differences in space with periodic boundary conditions.

Part 2 (20 points)

We choose $D_u = 2.0 \times 10^{-5}$ and $D_v = 10^{-5}$. Let $\Omega = [0, 2.5]^2$ denote the space domain and $\Omega_{\text{out}} = \Omega \backslash \Omega_{\text{in}},\ \Omega_{\text{in}} = [1.15, 1.35]^2$. Then the initial conditions are

$$u(\vec{x},0) = 1, \quad v(\vec{x},0) = 0, \qquad \text{on } \Omega_{\text{out}} \qquad (3)$$

$$u(\vec{x},0) = 1/2 + R/200, \quad v(\vec{x},0) = 1/4 + R/400, \qquad \text{on } \Omega_{\text{in}} \qquad (4)$$

where R is random noise sampled from a uniform distribution on $[-1,1]$. Using 256 grid points in each spatial direction and the time step $\Delta t = \frac{5}{6}\Delta x^2/(4D_u)$ run multiple simulations from $t = 0$

to $t = 10000$ using the values for κ and F as given in the table below and plot $u(x, y)$ at the time intervals $t = [1000, 2000, 5000, 10000]$. What do you observe?

κ	0.06	0.056	0.055
F	0.04	0.02	0.03