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Advanced Computational Science
SS 2009

Serie 9 Ausgabe: April 28, 2009 Abgabe: May 5, 2009

Guidelines

▷ Submission: Submit a written or typed report with your answers and graphics.

▷ Code: Send your source code (and makefile) to the TA of your session. Use "[ACS] Homework9" in the subject of your email.

▷ Credit: Your report should list all the contributors.

▷ Bugs: Print-outs of your source code are *not* required in your report, unless you have bugs. Help us give you partial marks.

▷ Deadline: Assignments have to be handed in at the beginning of the next exercise session.

Exercise 1 (40 points) Wine cellar problem

Wineries are interested in storing wine at a place where the temperature is optimal for wine maturation. In addition, the temperature variations should be minimal and slow. Over the years, mankind has figured out that in nature, such conditions are met underground. In the following exercise, we'll investigate the temperature variations in the ground using a simple model to see where best to store the wine casks.

For this task, we will use a one-dimensional diffusion equation

$$\frac{\partial T(z,t)}{\partial t} = D\frac{\partial^2 T(z,t)}{\partial z^2}.$$

Here $T(z,t)$ is the temperature at depth z ($z = 0$ is the ground surface and $z > 0$ denotes a depth below the ground surface) and at time t, and D is the diffusivity. The temperature at the ground surface, $T(0,t)$ is governed by some temperature cycle and is hence given by $T(0,t) = a + b\sin(\frac{2\pi}{\tau}t)$. The domain of our study stops at z_{max} where we assume that temperature does not vary anymore $\frac{\partial T}{\partial z}(z_{max}) = 0$. The initial conditions are $T(z,0) = a, \ \forall\, z$.

1. (15 points) Using an explicit-euler time-integration scheme and second-order finite differences in space scheme, implement a code to solve this diffusion equation using n grid points over $z \in [0, z_{\max}]$ up to a final time t_f.

2. Using the above code, simulate $T(z,t)$ up to a final time of 10 years, with $n = 501$ grid points over $z \in [0\,\mathrm{m}, 20\,\mathrm{m}]$ with $[D,\ a,\ b,\ \tau] = [1.43\mu\mathrm{m}^2/\mathrm{s},\ 12°\mathrm{C}, 10°\mathrm{C},\ 1\,\mathrm{year}]$.

   2.1. (10 points) On a single graph, plot T vs. z at t = 3, 3.25, 3.5, 3.75 and 4 years. Comment on your results.

   2.2. (10 points) On a single graph, plot T vs. t at z = 2, 3, 4 and 5 m. What happens to the temperature variations at large times.

2.3. (5 points) Plot the amplitude of temperature variation as a function of z at large times (over the period $t \in [3, 4]$). Using this information and assuming that the average temperature at the ground surface is optimal for wine maturation, at which depth closest to the ground surface would you store the wine casks if a variation amplitude of $2°\mathrm{C}$ is allowed?