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Advanced Computational Science
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Serie 8 Ausgabe: April 21, 2009 Abgabe: April 28, 2009

Guidelines

▷ Submission: Submit a written or typed report with your answers and graphics.

▷ Code: Send your source code (and makefile) to the TA of your session. Use "[ACS] Homework8" in the subject of your email.

▷ Credit: Your report should list all the contributors.

▷ Bugs: Print-outs of your source code are *not* required in your report, unless you have bugs. Help us give you partial marks.

▷ Deadline: Assignments have to be handed in at the beginning of the next exercise session.

Exercise 1 (40 points) Runge-Kutta integration

We consider the mass-spring problem. The mass m is connected to a spring with a constant k; its motion obeys the Ordinary Differential Equation

$$m\frac{d^2x}{dt^2} = -kx \; . \tag{1}$$

In this exercise we will consider the values $m = 1$ and $k = \pi^2$. The initial conditions are $x(0) = 1$ and $\frac{dx}{dt}(0) = 0$.

1. (5 points, paper and pencil) Cast the mass-spring problem as a system of first order ODEs (i.e. only involving first derivatives)

2. (10 points, paper and pencil) What are the eigenmodes of this system? How would you characterize its behavior? Discuss the stability of the Euler schemes (explicit and implicit) for such a system.

3. (10 points, paper and pencil) Study the linear stability of the fourth order Runge-Kutta schemes for the modes identified in part 2: what is the constraint on the time-step $\Delta t < \Delta t_{\text{critical}}$?

4. (15 points, programming) Implement the classical fourth order Runge-Kutta method (RK4C). For the problem $du/dt = f(u,t)$, the substeps of RK4C are given by

$$\begin{aligned}
k_1 &= \Delta t f(u^n, t^n)\\
k_2 &= \Delta t f(u^n + \frac{1}{2}k_1, t^n + \frac{1}{2}\Delta t)\\
k_3 &= \Delta t f(u^n + \frac{1}{2}k_2, t^n + \frac{1}{2}\Delta t)\\
k_4 &= \Delta t f(u^n + k_3, t^n + \Delta t)
\end{aligned}$$

and the whole step is then

$$u^{n+1} = u^n + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right) \ . \tag{2}$$

Apply your scheme to the mass-spring problem (part 1) and integrate it up to time $t = 20$. Investigate and verify the predicted stability: use $\Delta t = 1/8, 1/4, 1/2, 3/4, 1, 2$. Comment your results.