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Advanced Computational Science
SS 2009

Serie 7

Ausgabe: April 07, 2009 Abgabe: April 21, 2009

Guidelines

▷ Submission: Submit a written or typed report with your answers and graphics.

▷ Code: Send your source code (and makefile) to the TA of your session. Use ”[ACS] Homework7” in the subject of your email.

▷ Credit: Your report should list all the contributors.

▷ Bugs: Print-outs of your source code are *not* required in your report, unless you have bugs. Help us give you partial marks.

▷ Deadline: Assignments have to be handed in at the beginning of the next exercise session.

Exercise 1 Stability of ODEs (40 points)

Consider the sailing of a sailboat. Let $\mathbf{u} = [u, v]^T$ be the vector containing the x and y velocity components of the sailboat. Furthermore, we have a particular weather condition described by the vector of wind velocities $\mathbf{u_w} = [u_w, v_w]^T$. In this case, we can model the velocity evolution of the sailboat in time as an initial value problem with the following coupled system of ordinary differential equations:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} u(t) \\ v(t) \end{pmatrix} = \mathbf{A}\begin{pmatrix} u_w - u(t) \\ v_w - v(t) \end{pmatrix},$$

where $\mathbf{A}$ is a two-by-two coefficient matrix.

Suppose currently the wind vector is given by

$$\begin{pmatrix} u_w \\ v_w \end{pmatrix} = \begin{pmatrix} 10.0 \\ 3.0 \end{pmatrix},$$

and the matrix of coefficients is

$$\mathbf{A} = \begin{pmatrix} 1.0 & 0.0 \\ -0.3 & 0.2 \end{pmatrix}.$$

We are interested in the time period between $t_0 = 0$ and $t_{\max} = 15.0$. The initial velocity of the sailboat is zero.

▷ Part 1 (10 points) Consider the explicit and the implicit Euler timestepping schemes. What is the largest time step allowed by each of these two schemes to ensure stability for the current problem? (pen and paper)

▷ Part 2 (10 points) Implement both Euler schemes to solve for the above system of ODEs. Given the time frame we are interested in, analyse the solution for $dt = [3.0, 2.0, 1.0, 0.5, 0.1]$. Plot the time-evolution for both velocity components in one graph per scheme and comment on the results.

▷ Part 3 (10 points) Suppose the (1,1)-element of matrix $\mathbf{A}$ increases from 1.0 to 10.0 thanks to a breakthrough in sail technology. We get a new coefficient matrix $\mathbf{A}_{\text{new}}$:

$$\mathbf{A}_{\text{new}} = \begin{pmatrix} 10.0 & 0.0 \\ -0.3 & 0.2 \end{pmatrix}.$$

Repeat part 1 for with this new matrix. (pen and paper)

▷ Part 4 (10 points) Use your implementation of both Euler schemes to analyse the solution of the system of ODEs with the new matrix $\mathbf{A}_{\text{new}}$. Describe what the result looks like if you use $dt = 0.5$. In addition, plot the two components of the solution versus time using $dt = 0.01$. Explain how the behavior of the timestepping schemes changes for this case, as compared to your previous results.