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Advanced Computational Science
SS 2009

Serie 5 Ausgabe: March 24, 2009 Abgabe: March 31, 2009

Guidelines

▷ Submission: Submit a written or typed report with your answers and graphics.

▷ Code: Send your source code (and makefile) to the TA of your session. Use "[ACS] Homework5" in the subject of your email.

▷ Credit: Your report should list all the contributors.

▷ Bugs: Print-outs of your source code are *not* required in your report, unless you have bugs. Help us give you partial marks.

▷ Deadline: Assignments have to be handed in at the beginning of the next exercise session.

Exercise 1 (10 points) Gauss Quadrature

Given the Gauss quadrature points and weights (download the tables from the course webpage), evaluate:

$$I = \int_0^1 e^{-100x^2} dx \tag{1}$$

using $n = 2, 4, 6$ and 8. Plot and compare the errors with the errors of the same evaluation using Romberg integration for a number of intervals $n = 2, 4, 6$ and 8. Discuss the performance of these integration schemes and their respective advantages.

Exercise 2 (30 points) Jurassic Park

Predator-prey models are frequently used to describe the dynamics of ecosystems i.e. species sharing a common environment. Models of these systems were originally developed by Lotka and Volterra (1925) and generally consist of a system of nonlinear ordinary differential equations.

1. (20 points) In their original form, these models involve two species, x the prey (e.g. Diplodocus) population and y the predator (e.g. Tyrannosaurus Rex) population:

$$\frac{dx}{dt} = (A - By)x \tag{2}$$
$$\frac{dy}{dt} = (Cx - D)y \tag{3}$$

where A is the birth rate of species x, B is the predating rate of species x by species y, C is the birth rate of species y sustained by hunting species x, D is the death rate of species y and $A, B, C, D \geq 0$. Consider the following four initial conditions:

▷ $x(0) = 30, y(0) = 30$

- ▷ $x(0) = 100$, $y(0) = 30$
- ▷ $x(0) = 200$, $y(0) = 30$
- ▷ $x(0) = 200$, $y(0) = 20$

Integrate (using explicit Euler) the corresponding system of ODE's (200 time steps with a time step of 1.0) $A = 0.1$, $B = 0.005$, $C = 0.001$, $D = 0.2$ for the four different initial conditions and plot x and y versus time. Moreover, plot y versus x for all four initial conditions on *one single* graph. Comment on your observations.

2. (10 points) Based on this model, the prey population can increase exponentially if no predator is present in the environment. However, in reality, a population can not increase without limits due to many different causes referred to as the *social phenomena*, e.g. overcrowding. We will consider a predator-prey model with limited growth described by the following system of ODE's:

$$\frac{dx}{dt} = (A - By - \lambda x)x \quad (4)$$

$$\frac{dy}{dt} = (Cx - D - \mu y)y \quad (5)$$

The additional constants λ and μ describe the death of species x and y due to social phenomena.

Integrate (using explicit Euler) the corresponding system of ODE's (200 time steps with a time step of 0.001) using $A = 2$, $B = 1$, $C = 2$, $D = 3$, $\lambda = 1$, $\mu = 1$ with $x(0) = 100$ and $y(0) = 50$ and plot x and y versus time on the same graph. Moreover, plot y versus x. What is happening to the species of this environment?