Discrete vortex methods in bridge aerodynamics and prospects for parallel computing techniques

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ABSTRACT: The paper presents a short review of CFD applications in bridge aerodynamics with special attention to particle (vortex) methods. The paper includes an introduction to particle methods and their similarity and differences with the finite-difference methods. The focus is on two-dimensional problems, since the majority of the results presented in bridge aerodynamics so far are restricted to two dimensions. Some prospects for parallel implementation of two- and three-dimensional particle methods are outlined.

1 INTRODUCTION

Traditionally, bridge aerodynamics has been exercised in wind tunnels using section models and full bridge model tests. The wind tunnel tests require typically from one week to several months of measurements and analysis (Reinhold, Brinch, and Damsgaard 1992). Thus, as tool for benchmarking a suite of bridge section candidates, the wind tunnel is not the optimal tool.

In the last 10 years, Computational Fluid Dynamics (CFD) has emerged in civil engineering, including the field of bridge aerodynamics as a viable alternative to the preliminary wind tunnel tests (Larsen and Walther 1997a), and even for smaller bridges to replace wind tunnel tests.

The CFD methods applied range from "fully" commercial tools e.g., the finite-volume based CFX and STAR-CD, to dedicated special purpose programs e.g. finite-difference codes (Onyemelukwe and Bosch 1993), (Tamura, Itoh, Wada, and Kuwahara 1993), and (Fujiwara, Kataoka, and Ito 1993), finite-element codes (Nomura 1993) and (Shimura and Sekine 1993), and discrete vortex codes e.g., DVMFLOW (Walther 1994).

In the following we shall present a short overview of the particle (vortex) methods applied to bluff body flows and particular bridge aerodynamics. The focus will be on two-dimensional particle methods, since the majority of the present applications have been restricted to two dimensions.

The paper does not pretend to cover all of the different methodologies used in particle methods, neither their applications. For thorough review papers on particle methods the reader is referred to Leonard (1985), Sethian (1991), and Winckelmans and Leonard (1993) for recent advances in particle methods, and to Sarpkaya (1989) for an overview of applications.

When using robust numerical methods such as the particle (vortex) method one might distinguish between actual *solution* of the Navier-Stokes equations, or *modelling* of the large scale structures of the flow. The strength (and perhaps weakness) of the particle method(s) is it's ability to produce "flow-like" pictures and possibly even global quantities without actually modelling the correct physics or even solving the pertinent governing equation.

2 GOVERNING EQUATIONS

The governing equations for an incompressible, viscous fluid in a region $\mathcal{D}$ are the Navier-Stokes equations

$$\frac{D\vec{v}}{Dt} = -\frac{1}{\rho}\vec{\nabla}p + \nu\vec{\nabla}^2\vec{v} \quad \text{in } \mathcal{D} \tag{1}$$

$$\nabla\cdot\vec{v} = 0 \quad \text{in } \mathcal{D} \tag{2}$$

$$\vec{v} = \vec{v}_s \quad \text{on } \partial\mathcal{D} \tag{3}$$

where p is the pressure, $\vec{v}$ the fluid velocity, ν the kinematic viscosity, and D/Dt the material deriv-

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pressible, and (3) is the no-slip condition.

Taking the curl of (1) we obtain the vorticity transport equation

$$\frac{D\vec{\omega}}{Dt} = (\vec{\omega}\cdot\vec{\nabla})\vec{v} + \nu\vec{\nabla}^2\vec{\omega} \tag{4}$$

where $\vec{\omega} \equiv \vec{\nabla}\times\vec{v}$ is the fluid vorticity. The velocity is related to the vorticity by the Cauchy Riemann equations

$$\vec{\omega} = \vec{\nabla}\times\vec{v} \qquad \nabla\cdot\vec{v} = 0 \tag{5}$$

or by the Biot-Savart relation

$$\vec{v}(\vec{x},t) = -\frac{1}{4\pi}\int_{\mathcal{D}} \frac{\vec{\omega}\times\vec{\Delta}}{|\vec{\Delta}|^3}\,d\vec{y} + \frac{1}{4\pi}\int_{\partial\mathcal{D}} \frac{(\vec{v}_s\cdot\vec{n})\vec{\Delta} - (\vec{v}_s\times\vec{n})\times\vec{\Delta}}{|\vec{\Delta}|^3}\,d\vec{y} \tag{6}$$

where $\vec{\Delta} = \vec{x} - \vec{y}$, and $\vec{n}$ is the unit normal at the boundary. The surface integral accounts for the vorticity outside the domain $\mathcal{D}$ (e.g., the vorticity within a rotating body $\vec{\omega} = \vec{\omega}_s = 2\vec{\Omega}(t)$).

3 TWO-DIMENSIONAL PARTICLE (VORTEX) METHODS

Since engineering applications of the particle method have mostly focused on two-dimensional problems we shall restrict the following discussion to two dimensions. However, extension of the particle method to three dimensions is possible see e.g., Winckelmans and Leonard (1993), Kiya (1993), Almgren, Buttke, and Colella (1994), and Gharakhani and Ghoniem (1996).

In two-dimensional flow the vorticity transport equation (4) is a scalar equation ($\vec{\omega} = \omega\vec{e}_z$), thus

$$\frac{D\omega}{Dt} = \nu\nabla^2\omega \tag{7}$$

The particle methods approximate the vorticity field by a set of Lagrangian points $(\vec{x}^p, \Gamma^p)$

$$\omega_\sigma(\vec{x},t) = \sum_q \Gamma^q \zeta_\sigma(\vec{x} - \vec{x}^q(t)) \tag{8}$$

where Γ^p is the particle strength, and ζ_σ a smooth approximation of the Dirac delta function ($\delta(\vec{x}) = 1$, for $|\vec{x}| = 0$, and zero elsewhere). σ is the cut-off (core) radius. Different smoothing functions (similar to the various finite-difference schemes in the finite-difference methods) have been proposed in the literature cf. Beale and Majda (1985). The or algebraic functions

$$\zeta(\rho) = \frac{1}{2\pi}e^{-\rho^2} \tag{9}$$

$$\zeta(\rho) = \frac{1}{2\pi}\frac{2}{(\rho^2+1)^2} \tag{10}$$

$$\zeta(\rho) = \frac{1}{2\pi}\frac{4}{(\rho^2+1)^3} \tag{11}$$

and $\zeta_\sigma(\vec{x}) = 1/\sigma^2\zeta(|\vec{x}|/\sigma)$.

3.1 *Convection*

The particle approximation of (6) using (8) is

$$\vec{v}_\sigma(\vec{x}) = \sum_q \vec{K}_\sigma(\vec{x} - \vec{x}^q) \times (\Gamma^q\vec{e}_z) + \mathcal{O}\left((h/\sigma)^m\right) \tag{12}$$

where $\vec{K}_\sigma$ is the smoothed velocity kernel, $\vec{K}_\sigma(\vec{x}) = -(q_\sigma(\vec{x})/|\vec{x}|^2)\vec{x}\times$, and q_σ is the velocity smoothing as defined by ζ. h is the typical particle spacing, and m is a parameter related to the number of derivatives that exists of the smoothing function (for (9), (10), and (11), $m = \infty$). Thus, for the method to converge, the particles must be sufficient close (the cores must overlap)

$$\frac{\sigma}{h} > 1 \tag{13}$$

The "naive" implementation of (12) clearly requires $\mathcal{O}(N^2)$ operations, where N is the number of particles. For todays personal computers the maximum number of particles are below 10.000 in order to obtain reasonable computing times (one CPU minute per time step). "Fast methods" have been developed which require $\mathcal{O}(N\log N)$ operations cf. Barnes and Hut (1986) (tree code), Birdsall and Fuss (1969), and Christiansen (1973) (Cloud-In-Cell), or even $\mathcal{O}(N)$ cf. Greengard and Rokhlin (1987) (tree code), thus allowing simulating with $10^5 - 10^6$ particles cf. (Koumoutsakos 1993) and (Walther 1994).

Thus the governing equations for the convection problem (the Euler equations, $D\omega/Dt = 0$) reduce to a set of ordinary differential equations

$$\frac{d\vec{x}^p(t)}{dt} = \vec{v}_\sigma(\vec{x}^p, t) \tag{14}$$

$$\frac{d\Gamma^p(t)}{dt} = 0 \tag{15}$$

which are solved by standard Runge-Kutta or Adams-Bashforth methods. Using higher order time integration for (14) effectively removes any "numerical diffusion" from the method.

Convergence of the method for the Euler equation (14, 15) has been proved by Beale and Majda (1982) and Beale (1986).

Equations (14) and (15) describe the evolution of an inviscid fluid: vorticity (circulation) in the flow is carried with the local velocity without change in magnitude. In order to describe diffusion of vorticity one or both of the dependent variables ($\vec{x}^p$, Γ^p) are allowed to undergo a temporal change.

3.2.1 Random walk

The first successful application of viscous particle methods is due to Chorin (1973), in which he simulated diffusion by adding random motion (walk) to the particles. Thus, the particle position is modified according to

$$\vec{x}^p_{n+1} = \tilde{x}^p_{n+1} + \vec{\eta} \tag{16}$$

where $\vec{\eta}$ is a random number with zero mean and variance $2\nu\delta t$, and δt is the time step. $\tilde{x}^p_{n+1}$ is the particle position after the convection step at the $n+1$ time step, and $\vec{x}^p_{n+1}$ is the final particle position at $n+1$. The random walk method suffers from low convergence rate $\mathcal{O}\left((1/\mathrm{Re})(1/\sqrt{N})\right)$, where Re is the Reynolds number cf. (Milinazzo and Saffman 1977) and (Roberts 1985).

The appearance of the (1/Re) in the truncation error has lead to the questionable conclusion, that the random vortex method is capable of handling high Reynolds number flows per se. However, for direct numerical simulation of the Navier-Stokes equation, the random vortex methods (as well as other vortex methods and finite-difference methods) require sufficient spatial and temporal resolution. Thus, results obtained with "few" particles (or "few" mesh points), where "few" is closely related to the Reynolds number, should at best be considered as models of the Navier-Stokes equation, and not actual solution of the equations.

The random walk has the important feature of being insensitive to the particle spacing and it is suppose to "place the particles where they are needed". In particular, the creation of particles at the boundary secures reasonable resolution of the boundary layer without relying on re-meshing strategies. A large number of particle method implementations have used the random walk, cf. e.g., Smith and Stansby (1988), Gharakhani and Ghoniem (1996), Savoie, Gagnon, and Mercadier (1996), and Walther and Larsen (1997a).

3.2.2 Diffusion velocity method

The diffusion velocity method proposed by Ogami and Akamatsu (1991) gives the deterministic motion of particle paths (not constant vorticity) as

$$\vec{v}_d = -\frac{\nu}{\omega}\vec{\nabla}\omega \tag{17}$$

The singularity at $\omega \to 0$ is circumvented by observing that

$$\vec{v}_d = -\nu\vec{\nabla}(\ln|\omega|) \tag{18}$$

Thus (14) is replace by

$$\frac{d\vec{x}^p(t)}{dt} = \vec{v}_\sigma(\vec{x}^p, t) + \vec{v}_d \tag{19}$$

to include the effect of diffusion. See Walther (1993), Strickland and Wolfe (1996), and Marshall and Grant (1997) for recent applications of the method.

3.2.3 Particle strength exchange method

The Particle Strength Exchange (PSE) method (Degond and Mas-Gallic 1989) approximates the Laplacian ($\nabla^2\omega$) with an integral operator, thus requiring temporal development of the particle strength

$$\begin{aligned}\frac{d\Gamma^p}{dt} &= \frac{2\nu}{\sigma^2}\sum_q \eta_\sigma(\vec{x}-\vec{x}^q)(vol^p\Gamma^q - vol^q\Gamma^p) \\ &+ \mathcal{O}\left(\frac{1}{\mathrm{Re}}\left((\sigma/L)^r + (h/\sigma)^m/(\sigma/L)\right)\right)\end{aligned} \tag{20}$$

where L is the global length scale, r is related to the moment properties of the smoothing function (for (9) and (11), $r = 2$, and (10), $r = 0$), and vol is the particle volume (constant for incompressible flows). The η_σ is a smoothing function (not to be confused with the random walk), approximating the heat kernel. Thus, a natural choice is the Gaussian

$$\eta_\sigma(\vec{x}) = \frac{1}{2\pi\sigma^2}e^{-|\vec{x}|^2/2\sigma^2} \tag{21}$$

Since $\sigma/L \ll 1$, the PSE method clearly requires strict adherence to the particle overlap, and in particular since the temporal development of the particle strength is directly affected. The method has be successively applied to a number of fluid flow problems cf. Winckelmans (1989), Cottet (1991), Koumoutsakos and Leonard (1995), and Koumoutsakos (1997).

3.2.4 Direct differentiation

In the simulation of the flow past a sphere, Fishelov (1990) evaluated the Laplacian directly by differentiating (8), thus

$$\nabla^2\omega_\sigma(\vec{x}) \approx \sum_q \Gamma^q\nabla^2\zeta_\sigma(\vec{x}-\vec{x}^q) \tag{22}$$

Gallic and Raviart 1987).

3.2.5 Particle strength re-distribution

Finally, Shankar and van Drommelen (1996) proposed a modification to the PSE scheme, in which the particle strength is re-distributed between the particles while satisfying the vorticity moments. The scheme is conservative but requires CPU time of the same order of magnitude as the fast method used for convection (van Dommelen and Rundensteiner 1989).

3.3 *Boundary conditions*

The fundamental boundary condition is the no-slip velocity boundary condition (3), which is enforced via the surface vorticity given by the kinematic constrains imposed by the Biot-Savart equation Wu (1975) and (1976). When evaluated at the boundary, equation (6) poses a boundary integral problem, which in two dimensions is non-unique due to the multiply connected domain. A unique solution can be found imposing the conservation of total vorticity (Kelvin's theorem) in a least square formulation cf. e.g., Walther and Larsen (1997a) or by removing the singularity of the kernel (eigenvalue problem) as proposed by Koumoutsakos and Leonard (1993).

Uhlman and Grant (1993) proposed a combination of source and vortex sheets to obtain higher convergence for the boundary element equations.

For single body configurations or multiple bodies moving *en bloc*, the dense influence matrix needs only to be inverted once. Thus, the solution of the boundary element problem typically requires less than 5 % of the total CPU time. Indeed, the solution of the boundary element problems is probably one of the main reason for the early success of the particle method, since it secures the satisfaction of fundamental Kelvin theorem.

Now, depending on the applied diffusion scheme, the created surface vorticity or surface vorticity flux is handled differently.

3.3.1 Random walk

For the random walk method, one-sided random numbers are used at solid boundaries to diffuse the particles into the flow. The pdf is

$$P(n) = \frac{1}{\sqrt{\pi\nu\delta t}} e^{-n^2/4\nu\delta t}, n > 0 \tag{23}$$

Particle entering the solid (by random walks) are usually removed (Chorin 1978). Smith and where the particles are reflected or absorbed during one time step before re-diffused, now with a modified random walk

$$Q(n) = \frac{\sqrt{\pi}}{\sqrt{4\nu\delta t}} \left[1 - \mathrm{erf}\frac{n}{\sqrt{4\nu\delta t}} \right], n > 0 \tag{24}$$

3.3.2 Particle strength exchange

The PSE scheme usually relies on the vorticity flux in terms of the slip velocity or vortex sheet ($\gamma = 2v_{slip}$) occurring after the convection step

$$\nu\frac{\partial\omega}{\partial n} = -\frac{\partial\gamma}{\partial t} \tag{25}$$

Thus, the diffusion of the particles in the fluid are modified to account for this surface vorticity flux as

$$\begin{aligned} \frac{d\Gamma^p}{dt} &= \frac{2\nu}{\sigma^2}\sum_q \eta_\sigma(\vec{x} - \vec{x}^q)(vol^p\Gamma^q - vol^q\Gamma^p) \\ &+ \frac{2\nu}{\sigma^2}\sum_m^M H(x^p, x^m)\frac{\partial\omega}{\partial n}(x^m), \end{aligned} \tag{26}$$

where M is the number of boundary panels, and H is diffusion kernel for a finite vortex sheet

$$H(x,y) = \frac{e^{-y^2/4\nu\delta t}}{\sqrt{4\nu\delta t}} \left[\mathrm{erf}\frac{x-d}{\sqrt{4\nu\delta t}} + \mathrm{erf}\frac{x+d}{\sqrt{4\nu\delta t}} \right] \tag{27}$$

and $2d$ is the length of the vortex sheet, erf is the error function, and y is in the wall normal direction cf. (Koumoutsakos, Leonard, and Pépin 1994).

3.4 Aerodynamic forces

3.4.1 Pressure forces

The aerodynamic forces are efficiently computed from the surface vorticity flux

$$\frac{1}{\rho}\frac{\partial p}{\partial s} = -\nu\frac{\partial\omega}{\partial n} \tag{28}$$

and using (25), from the rate-of-change of the surface vortex sheet

$$\frac{1}{\rho}\frac{\partial p}{\partial s} = \frac{\partial\gamma}{\partial t} \tag{29}$$

The "effective" rate-of-change of the vortex sheet strength depends on the diffusion scheme cf. e.g. (Walther and Larsen 1997b). The total pressure forces are found integrating (29) along the solid boundary. The surface shear forces are normally negligible in bluff body aerodynamics, but can be computed from the surface vorticity as evaluated from (8).

The total aerodynamic forces are computed from the integration of the surface pressure and shear forces, or from a global momentum balance in terms of the rate-of-change of the first vorticity moment

$$\vec{F} = -\rho \frac{d}{dt} \int\int_{\mathcal{D}} \vec{x} \times \vec{\omega} \, \mathrm{d}\vec{y} \qquad (30)$$

Similarly, the aerodynamic moment is computed from the second vorticity moment (Wu 1981).

4 BLUFF BODY AERODYNAMICS

Relatively few simulations of bridge aerodynamics have to the authors knowledge been reported in the literature. Thus, in the following we shall include some "bluff body" simulations pertinent to numerical bridge aerodynamics.

Sarpkaya and Kline (1982) presented experimental data and discrete vortex simulations for the aerodynamic loads on four impulsively-started cylinders (circular, "D-shaped", "T-shaped", and a flat plate) at Reynolds numbers Re = 20000 – 30000. Inviscid vortex sheets were created and shed at the points of separation, which were determined from steady or unsteady momentum equations cf. (Sarpkaya and Schoaff 1979). Various ad-hoc mechanisms were devised to reduce the circulation in the wake. Reasonable lift and drag values were obtained for the circular cylinder, whereas larger deviations were observed for the bluff sections.

In the experimental study of the effect of fairings on the aerodynamic stability of bridge sections, Nagao, Utsunomiya, and Manabe (1993) also presented the aerodynamic loads using an inviscid discrete vortex method. Shedding vortices from the separation point only, excellent agreement was obtained with the experimental data.

Inamuro, Adachi, and Sakata (1993) studied the aeroelastic instability of a square cylinder using a viscid-like vortex method for a single degree-of-freedom structural system. Diffusion was modelled using the core spreading method cf. e.g. (Leonard 1980). The results were considered in qualitative agreement with experimental data.

Koumoutsakos and Shiels (1996) used the PSE scheme and the "fast method" by Carrier, Greengard, and Rokhlin (1988) to study the flow normal to a flat plate undergoing accelerating translational motion at Reynolds numbers 20–1000. For the accelerating plate, the numerical results confirmed previous experimental evidence of Kelvin-Helmholtz instability of the shear layers.

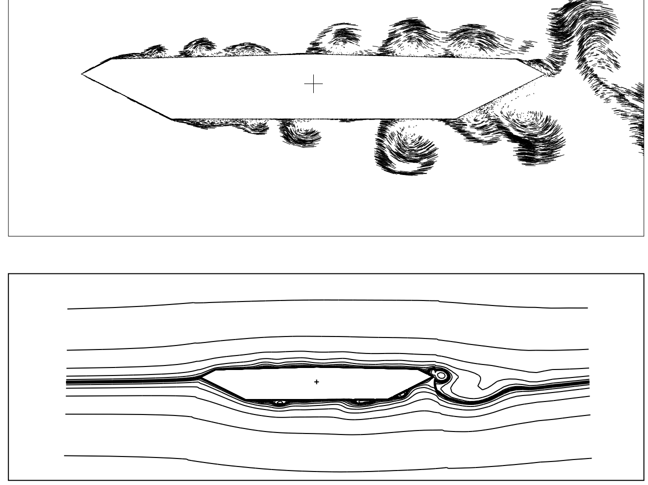

Figure 1: Discrete vortex method simulation of the flow past the Great Belt East bridge at Reynolds number 10^5 (Walther 1994). Top: position of particles in the vicinity of the bridge section; Bottom: Streamlines for bridge section undergoing pitch motion.

Walther (1994) modelled the flow past the Great Belt East bridge using a boundary element method, the random walk for diffusion and the "fast method" by (Carrier et al. 1988) cf. Figure 1. The static forces on the bridge section excluding the effect from the railing and crash barriers modelled experimentally were found in reasonable agreement with experimental data cf. Figure 2. Using forced oscillations, the aerodynamics derivatives were extracted for a range of reduced wind speeds ($U_R \leq 10$). From these, the critical wind speed for the erection stage of the bridge was found to 35 m/s in surprising good agreement with the wind tunnel value of 38 m/s. The excellent results generally obtained for moving bodies were ascribed to the two-dimensionality and strong forcing of the flow imposed by the motion of the body.

Later, the same code was modified to allow multiple structures and arbitrary solid body motion of the individual bodies (Walther and Larsen 1997a). Here, the flow past a flat plate with an oscillating trailing edge flap was studied and the aerodynamic loads compared with the inviscid forces and moments as given by Theodorsen (1935). At Reynolds number 10.000 and small amplitudes of motion excellent agreement was obtained.

The code has been further validated for several bluff body section including numerous bridge sec-

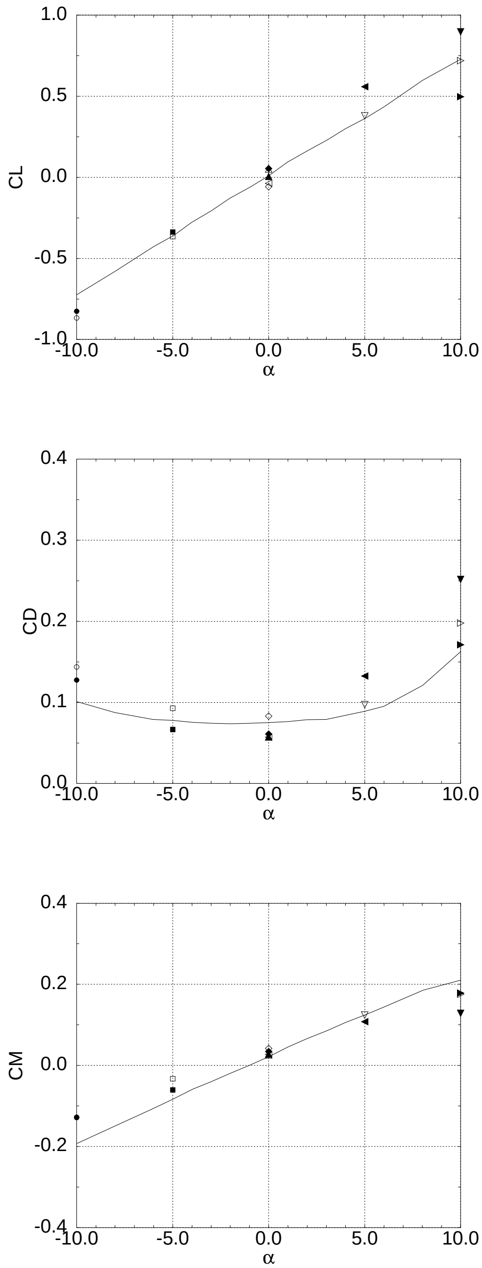

Figure 2: Static aerodynamic forces versus angle of attack for the Great Belt East bridge. — wind tunnel data Reinhold et al. (1992); symbols: discrete vortex method Walther (1994).

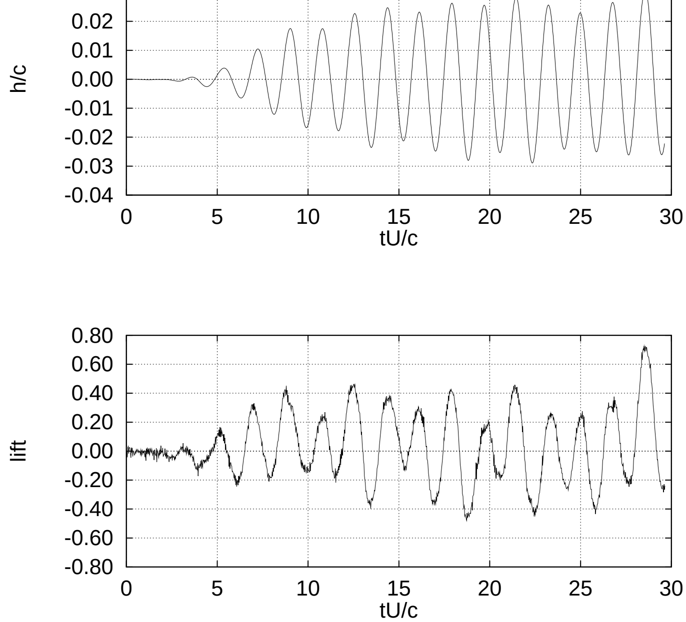

Figure 3: Simulated time history of the vertical position and non-dimensional lift force for the First Tacoma Narrows bridge (Larsen and Walther 1996).

tion cf. Larsen and Walther (1996, 1997, 1997a, 1997b). In (Larsen and Walther 1996) the vortex induced response of the First Tacoma Narrows bridge was simulated using an elastically suspended model cf. Figures 3 and 4. The peak amplitude of $h/c = 0.02$ compared reasonable with the experimental data available of $h/c = 0.024 - 0.040$ cf. (Farquharson 1952).

5 PARALLEL COMPUTING TECHNIQUES

Since particle methods involve arithmetic on large vectors, the method is in general ideally suited

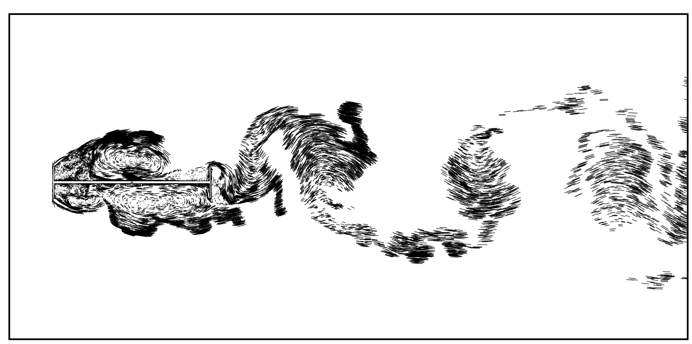

Figure 4: Instantaneous particle velocity at the position of the particles in the vicinity of the First Tacoma Narrows bridge (Larsen and Walther 1996).

long-range influence $(1/r)$ of the Biot-Savart equation does not allow straight forward identification and utilisation of "near-neighbour" particles for easy and efficient parallel implementation.

The "fast codes" do utilise the spatial distribution of the particle to compute the influence from distant particles using multipole and Taylor series expansions. However, the adaptivity of the particles require careful design of the underlying data structure to secure good load balance cf. e.g. (Salmon, Warren, and Winckelmans 1994).

A parallel implementation of the $\mathcal{O}(N^2)$ algorithm for the three-dimensional particle method is currently being developed. The following Section outlines the proposed algorithm and preliminary results.

5.1 $\mathcal{O}(N^2)$ algorithm

5.1.1 Load balance

For efficient explicit parallelisation, the particles and body surface points are distributed equally onto the processors. Furthermore, adding or removing particles during re-meshing (PSE scheme), or during particle creation and deletion at the boundary (random walk), requires the new particle configuration to be re-distributed in order to secure good load balance. Recursively sorting the processors in order of increasing number of particles, and allowing processors in the sorted list to interchange particles has proved efficient.

5.1.2 Particle interactions

Irrespectively of the physical arrangement of the processors, the particles are mapped onto the processors using a ring topology. The evaluation of the Biot-Savart equation (including stretching and diffusion) proceeds by making a copy of the particles (effectively a second ring), and allowing the the copy to rotate in order to collect half of the processor-processor interactions. At the end of the rotation, the collected particle velocity (and the rate-of-change of the particle strength) is returned to the origin and added to the results collected by the "stationary" processors. The mapping thus utilise the symmetry of the governing equations halving the CPU requirement.

5.1.3 Preliminary parallel results

Figure 5 shows the parallel speed-up on a 32 processor HP/Convex Exemplar for the simulation of a three-dimensional vortex ring without solid boundaries and without re-meshing. The code is written in FORTRAN 90 with explicit message passing using MPI. Convection is handled using second order Runge-Kutta and Adams-Bashforth time integration and the PSE scheme for diffusion. Excellent speed-up is observed for less that 15 processors. The decrease in speed-up occurring at 15 processors is due to the partitioning of the hardware (2×16 processors).

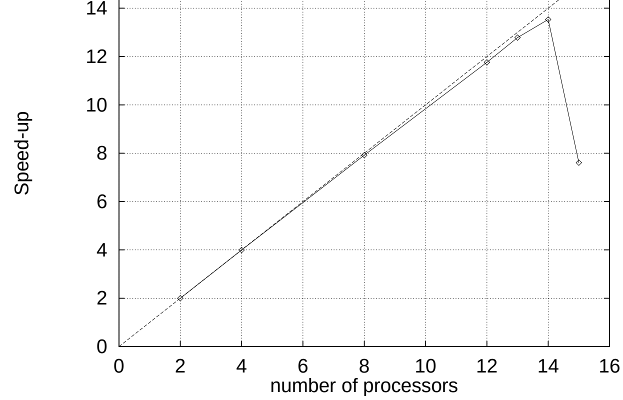

Figure 5: Parallel speed-up for the present three-dimensional $\mathcal{O}(N^2)$ particle method on a Convex/HP Exemplar using explicit message passing (MPI).

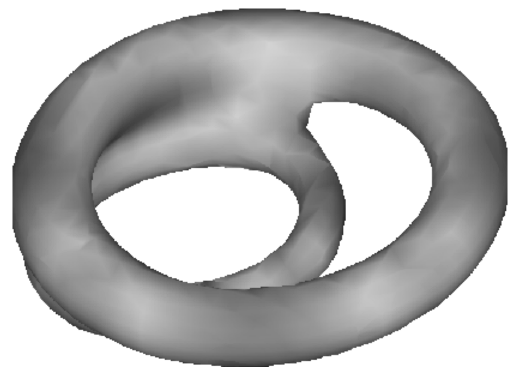

Figure 6: Two-ring configuration using the present three-dimensional $\mathcal{O}(N^2)$ particle method. The divergence of vorticity is shown on a constant vorticity surface.

Figure 6 shows the iso-vorticity value of the interaction of two vortex rings using the present three-dimensional code. Work is in progress to include solid walls.

5.2 Fast codes

A number of "fast codes" have been implemented for parallel computer architectures e.g., Salmon and Warren (1993), Salmon et al. (1994), Salmon

Warren, Leonard, and Jodoin (1996) using the $\mathcal{O}(N \log N)$ Barnes Hut algorithm.

Parallel implementations of the $\mathcal{O}(N)$ fast multipole method include Leathrum Jr. and Board Jr. (1992), Elliott and Board Jr. (1994), Gray (1994), Rankin and Board Jr. (1995), and Singer (1995). Turkiyyah, Reed, and Yang (1996) and Turkiyyah and Reed (1997) implemented the "fast" algorithm by Anderson (1992).

At present some of the three-dimensional parallel codes allow for simulations with 10^6 particle within 1–2 CPU minutes per time step making three-dimensional particle methods a viable tool for the study of bluff body aerodynamics.

The reader is referred to the references for details regarding the implementations and speed-up.

6 CLOSURE

We have presented a short review of the applications of particle (vortex) methods to bluff body aerodynamics. Current state-of-the-art schemes for convection, diffusion and the pertinent boundary conditions have been outlined and compared with finite difference methods.

The relative success of the particle method for predicting bridge aerodynamics including static forces, critical wind speed, and vortex shedding has been demonstrated for the streamlined Great Belt East Bridge and for the bluff body section of the First Tacoma Narrows Bridge.

Prospects for parallel implementation of three-dimensional particle methods including some preliminary results for the interaction of vortex rings has been presented.

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