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Detection of gravitational waves from coalescing binaries in the time domain

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Abstract. We propose a multistep procedure for the on-line detection and analysis of gravitational wave signals emitted during the coalescence of compact binaries. This procedure, based on a hierarchical strategy, consists of a rough analysis of the gravitational wave signal using adaptive line enhancers (ALE) filters and the controlled random search (CRS) optimization algorithm followed by a refined analysis using the classic matched-filtering technique. The results of simulations for the rough analysis are quite promising both for the relatively small computational power needed and for the robustness of the algorithms used, so that it could be very helpful for gravitational wave detection with very large baseline interferometric detectors like LIGO and VIRGO.

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1. Introduction

Coalescing compact binaries are probably the best candidates for gravitational wave (GW) detection with the new generation of long baseline interferometric antennas like LIGO [1] and VIRGO [2]. But, even if these interferometers seem to be sensitive enough for the detection of these sources, nevertheless GW signal analysis is still an open problem, the solution to which requires an appropriate selection of the data analysis techniques taking into account the shape of the expected signal, the noise of the detector and the available computing power [3, 4]. Up to now, the most frequently used algorithm for such a detection is the matched-filtering technique [5], well known in communication theory as the Wiener–Komolgorov optimum filter [6], which correlates the detector output with a template of the expected signal (matched filter). But, although very simple in principle, this technique requires a practically exact theoretical knowledge of the shape of the expected signal as a function of the unknown parameters describing the coalescing binary and, then, the correlation of the detector output with several thousands of templates. As is well known, these two requirements are very difficult to satisfy for coalescing binary signals [7, 8]. Moreover, the computing power needed for an on-line analysis is very high (at least of the order of tens of GFlops) and large parallel computers are necessary. Of course, the analysis

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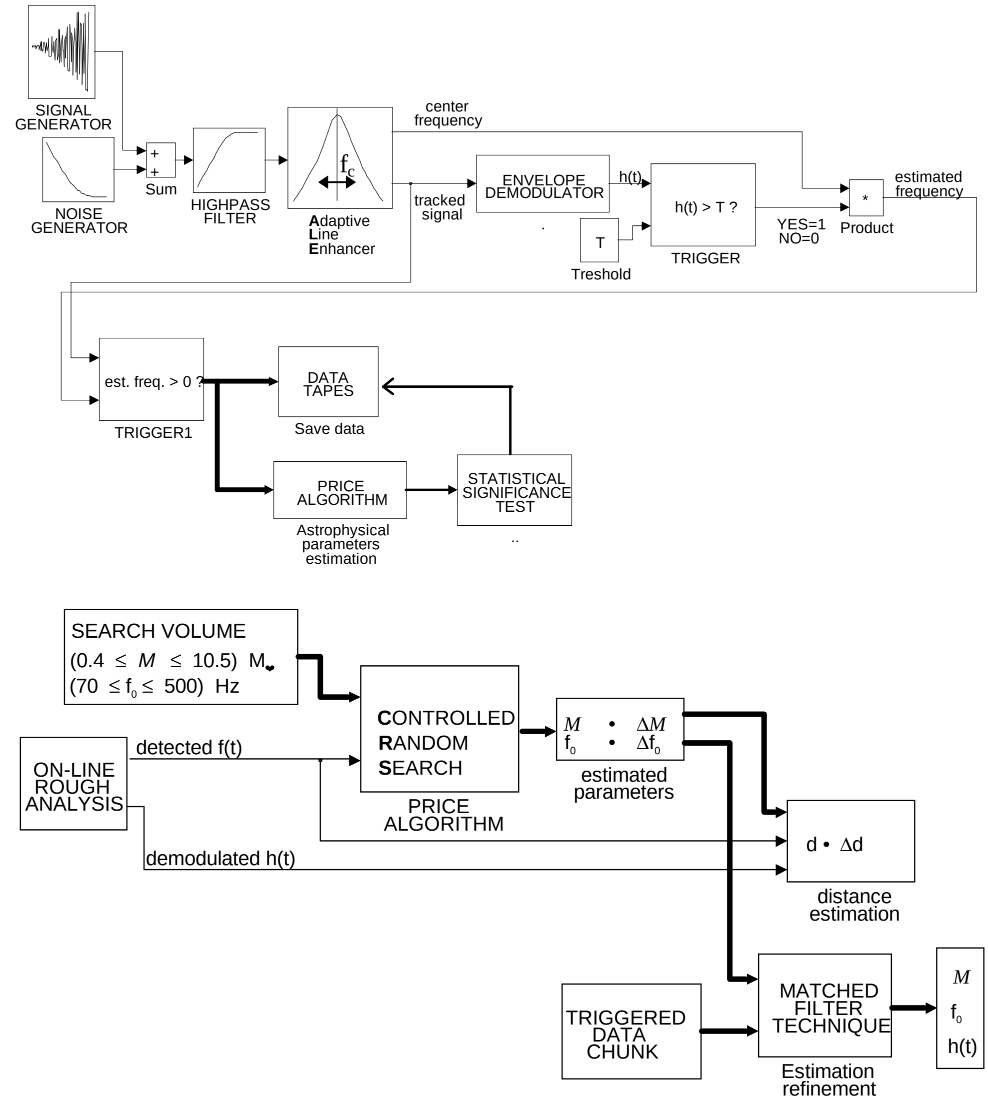

Figure 1. Scheme of the recursive centre-frequency adaptive filter we used for chirp tracking.

of these data could be done off-line, but an on-line selection of data frames would be better if a GW signal is present.

Thinking that a hierarchical strategy is the best way to overcome this problem, we have developed a procedure for a full on-line analysis which saves computing power but, at the same time, leaves a necessary uncertainty margin between the theoretically predicted signals and the experimental ones, without degrading the results of the data analysis. This procedure involves two steps (see figure 1).

(i) A rough analysis of interesting sequences of GW antenna output data with suitable algorithms (adaptive filters and global optimization algorithms).

(ii) Refined on-line data analysis with more powerful but computationally more complex algorithms (matched filters).

In particular, the rough analysis, like the refined one, must be really efficient and robust against false alarm detection and must not lose signals which can be extracted with refined algorithms. The strategy adopted for the rough analysis consists in dividing the problem of filtering into two parts in order to obtain separate information on the amplitude and frequency of the signal [9, 10]. To fulfil these requirements we tested an adaptive line enhancer algorithm (ALE) (adaptive filter) [11] coupled to the controlled random search (CSR) [12] (global optimization algorithm).

2. Adaptive line enhancers (ALE)

The algorithms we tested belong to the class of partially adaptive filters used to obtain a time–frequency output from the filtered signals. For this task we modified a classical filter which is used for the tracking of signals in additive white Gaussian noise (AWGN) to provide the characterization of the frequency or of the amplitude of the input signal in the *time domain*. We tested a classical infinite impulse response (IIR) based ALE filter, designed to be only partially adaptive by allowing a constrained recursive centre frequency adjustment in order to enhance noisy bandpass signals [11].

To test the efficiency and robustness of this algorithm, we simulated a simplified output sensitivity response of the VIRGO antenna between 1 Hz up to 5 kHz, taking account of the two main sources of noise in VIRGO, i.e. the low-frequency thermal pendular noise and the high-frequency shot noise [2, 13]. All the simulations were performed with additive prewhitened Gaussian noise (ApWGN) of different spectral linear densities, $\tilde{h} = 2 \times 10^{-21}$ Hz$^{-1/2}$ at 100 Hz (worst case) and $\tilde{h} = 2 \times 10^{-23}$ Hz$^{-1/2}$ at 100 Hz (best case).

We had to choose the optimum threshold for the implementation of the trigger. Using the ALE filter, the demodulator output power can be modelled in the presence of noise with a random variable having a Rayleigh distribution, with variance $[2(1-r)/(1+r)]\sigma_n^2 \equiv \sigma_N^2$, while the output power statistic becomes a *Rician* distribution, with mean $\mathcal{E}$ and variance σ_N^2 in the presence of a tracked signal. Assuming independent samples and a very low probability of the presence of a signal, that is $P_s \ll 1$, then the error probability, P_e, can be conveniently bounded as

$$P_e \leqslant 2\mathrm{e}^{-T^2/2\sigma_N^2} \tag{1}$$

Table 1. Numerical tests with ALE. The systems were chosen to cover the known range of the mass parameter, $\mathcal{M}$. In the table the headings refer to the component type of the coalescing system, the masses of the objects, the theoretical mass parameter, $\mathcal{M}_t$, the starting theoretical frequency, f_{0_t}, considered in the simulation, the theoretical duration, τ_t, of the coalescence in seconds, the theoretical distance, d_t, in Mpc; the corresponding estimates obtained from the analysis are shown with their standard errors with indices m.

System		Theoretical values				Measured values			
Type	$m_1 \div m_2$ $(M_\odot)$	$\mathcal{M}_t$ $(M_\odot)$	f_{o_t} (Hz)	τ_t (s)	d_t (Mpc)	$\mathcal{M}_m$ $(M_\odot)$	f_{o_m} (Hz)	τ_m (s)	d_m (Mpc)
NS–NS	$1.4 + 1.4$	1.21	100	2.15	500	1.3 ± 0.1	100 ± 5	1.80 ± 0.30	300 ± 40
NS–BH	$1.4 + 10.0$	2.99	100	0.49	500	3.00 ± 0.04	99.2 ± 0.7	0.46 ± 0.02	510 ± 70
BH–BH	$10.0 + 10.0$	8.70	100	0.082	500	7.72 ± 0.03	96.9 ± 0.2	0.08 ± 0.02	500 ± 100

which allowed us to evaluate the optimum threshold, T, in a closed form

$$T \geqslant \sqrt{-\frac{4(r-1)}{r+1}\ln\frac{P_e}{2}}\sigma_n. \qquad (2)$$

In our tests we assumed $P_e \cong 10^{-12}$ (corresponding to a false alarm probability of one per year), $r = 0.99$ and $\sigma_n = 2 \times 10^{-23}$ $\mathrm{Hz}^{-1/2}$. We obtained a threshold for the demodulated amplitude $h(t)$ equal to $T \cong 1.5 \times 10^{-23}$. This result is a direct consequence of the enhancement of the SNR performed by ALE which permits use of a threshold, T, for the output $\sqrt{(1+r)/[2(1-r)]}$ times smaller than the one necessary for the input signal [9].

We tested the performances of this algorithm within the procedure shown in figure 1 on the systems reported in table 1, using the chirp waveforms derived by Królak [14] and adapted for numerical computations by Verkindt [13] for the simulations.

In figure 2 the detected frequencies and demodulated amplitudes of a simulated signal in the time domain (full curve) and the theoretical trend of the amplitude and frequency (broken curves) of the coalescing systems emitting GWs at the Newtonian order along the time are shown for the system $1.4 + 1.4M_\odot$. In figure 3 the results of the detection of the above system obtained from the analysis of the coalescence parameters are shown: the histogram of the mass parameters distribution obtained by CRS (top), a normality test for the observed $\mathcal{M}$ distribution (middle) and, finally, a confidence ellipse at the two-σ level for f_0 versus $\mathcal{M}$ (bottom). In particular, using the CRS we found estimates of the parameters of the coalescing binary at the Newtonian order with relative errors less than 10% with respect to the theoretical values of the mass parameters (see table 1). The maximum relative error on the mass parameter is that of the system characterized by $\mathcal{M}_t = 0.44$ ($e_{\mathcal{M}} \approx 34\%$). The distances were also estimated, obtaining relative errors of less than 20%. To test the

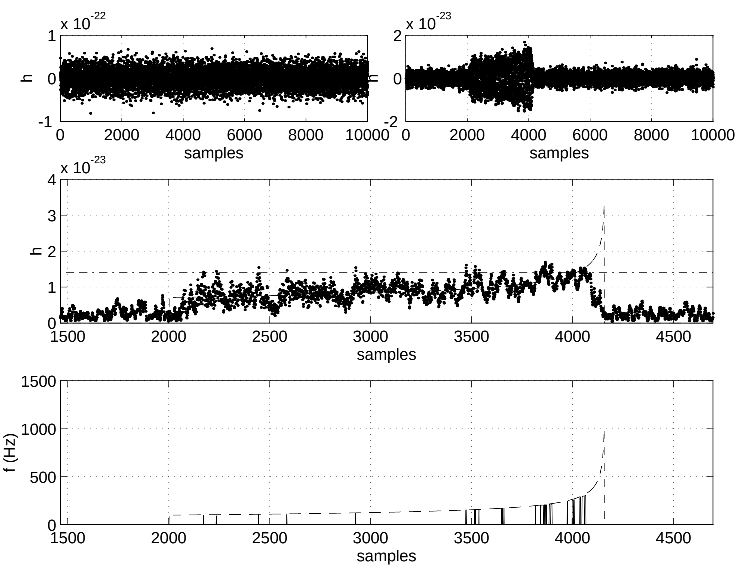

Figure 2. From the top to the bottom clockwise: the GW input signal buried in noise, emitted by the classical NS–NS coalescing binary system of $(1.4 \div 1.4M_\odot)$ and coming from a distance of 500 Mpc (SNR ≈ -3 dB); the output signal from ALE (note the improvement of the SNR$_{out}$ that reaches the value of $\approx +8$ dB); the demodulated amplitude, the threshold (broken curve) and the output frequency of the signal with respect to the theoretical one (broken curve) as a function of the time (the signal was applied after 2 s) are shown in the lower two figures.

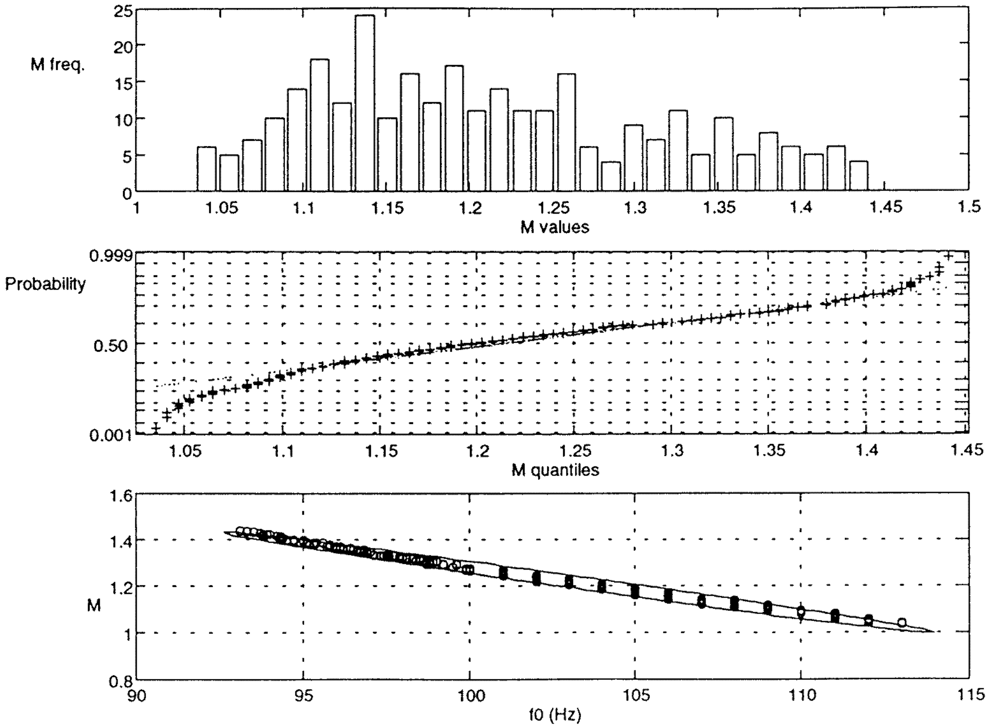

Figure 3. The output distribution of computed mass parameters of the binary by the CRS (top), normality test for the computed mass parameters, $\mathcal{M}$ (middle), the 95% confidence ellipse for the parameters solution $\mathcal{M}$ and f_0 (bottom).

statistical significance of the obtained results, we devised the following procedure. We generated a noise-only chunk of data with the same estimated duration of the simulated signal and we fed ALE with this *signal*. From the output frequencies thus obtained we run the CRS followed by a t-Student test between the set of parameters obtained from the suspected signal and the pure noise signal one. We performed a two tail null-hypothesis test using t-Student statistics for the mean at a 5% significance level to verify if the samples of mass parameters obtained from the simulated signals could be extracted from the same population of the mass parameters obtained from a pure noise signal. The results we got for the worst case noise environment, i.e. $\tilde{h} = 2 \times 10^{-21}$ Hz at 100 Hz, are practically the same as for the optimistic case apart from the limit of detection distance, which undergoes a scaling by a factor of 100 with respect to the optimistic sensitivity case.

3. Discussion and conclusions

We want to stress the following results:

(i) The adaptive filter we tested was very simple and of practically negligible computational complexity: the implementation of a single ALE requires 30 arithmetic operations. This class of filter can be easily used on-line with very effective performance.

(ii) Using a coupled ALE and CRS algorithm we can completely characterize the signal because we have two pieces of information concerning both the trend of the frequency and of the amplitude evolution of the coalescence signal in the time domain with input SNR ratios that can also be of the order of 0.2 (see figures 2 and 3). In particular, it is possible to obtain an on-line rough, but fast, estimation of the parameters of the coalescence using only an almost *exact* knowledge of the signal waveform, which can be very useful for the initialization of a more powerful refined analysis (matched filters) also on-line.

(iii) In our simulations we restricted the intervals of possible mass parameters so that the computational power demand for matched filters is lowered in such a way that could be easily feasible for an on-line analysis by a parallel computer. In fact, if we apply the hierarchical procedure using the results reported in table 1 obtained with ALE to a matched filtering technique, we reduce the computing power needed by a factor of 10 for the worst case (first system in table 1) and to a few tens of MFlops for the best case (last system in table 1).

Of course, at the moment there are limits and drawbacks, but we are confident that we will succeed in finding good solutions to the remaining problems both for ALE and CRS.

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