Flow Optimization Using Stochastic Algorithms

Petros Koumoutsakos and Sibylle D. Müller

Institute of Computational Science, ETH Zürich, CH-8092, Switzerland
petros@ethz.ch

Abstract. We present a set of stochastic optimization strategies and we discuss their applications to fluid mechanics problems. The optimization strategies are based on state-of-the-art stochastic algorithms and are extended for the application on fluid dynamics problems. The extensions address the question of parallelization, strategy parameter adaptation, robustness to noise, multiple objective optimization, and the use of empirical models. The applications range from burner design for gas turbines, cylinder drag minimization, aerodynamic profile design, micromixer, microchannel, jet mixing to aircraft trailing vortex destruction.

1 Introduction

The optimization of physical processes for applications in areas such as turbomachinery, aeronautics, and microtechnology poses different challenges to the optimization engineer.

Jet mixing, for example, is an application in which an increased mixing is aimed at in order to reduce noise, suppress signature, and increase lift in civil and/or military aircraft. Mixing the hot jet gas with the surrounding air can be enhanced by actuating the flow at the outlet of the jet. For a systematic search for optimal actuation parameters, one first needs to obtain the mixing rate as a function of the actuation parameters. This can be achieved by setting up experiments or simulations. Let us assume that a computer program is available that simulates the jet and computes the value of the mixing rate. Then, for an automated optimization, we need to wrap a search algorithm around the simulation program. What is a suitable optimization method in this case?

The program can compute the mixing rate as a function of the actuation parameters, that means, only function information is available but no gradient information. To obtain gradients, we would need to (i) approximate them using e.g. finite difference methods that are inaccurate [4], or (ii) compute them using automatic differentiation techniques which is not so trivial and might not be easily applicable to this particular case [3], or (iii) determine them using e.g. an adjoint variable method which is a difficult task that has not been solved yet for jet mixing (and for most other fluid dynamics applications) [36]. The problem of jet mixing exemplifies the difficulties associated with optimization using gradient based methods, namely noise, multiple minima, and the overall absence of an explicit function relating parameters and objective function. The same underlying difficulties are present in other applications tackled in this paper ranging from realistic turbomachinery to shape optimization in nanoscale structures. Therefore, the applicability of gradient methods in these case appears not convincing. More promising for these problems are nongradient optimization techniques despite their slow convergence properties. Among those, we can decide for deterministic or stochastic techniques. As the jet and other fluid dynamics applications represent highly dynamical systems that are susceptible to small changes in actuation parameters, a stochastic optimization algorithm that can handle noise is preferable.

Among stochastic search methods, evolutionary algorithms have become more and more popular in recent years, mainly because of their ease in implementation and their advantages compared with traditional algorithms especially when dealing with nondifferentiable, discontinuous, multimodal and/or noisy optimization functions. As most engineering optimization problems deal with such kinds of functions, it is obvious that evolutionary algorithms are an interesting alternative to classical methods.

Our stochastic optimization framework includes

- the Covariance Matrix Adaptation Evolution Strategy (CMA-ES) [19, 20, 31],
- Evolutionary Algorithms using adaptation techniques that employ Self-Organizing Maps [6, 7, 8, 23],
- Evolutionary Multi-Objective optimization algorithms with adaptation and noise-tolerance [5],
- Clustering Genetic Algorithms for finding correlations in a set of solutions [27], and
- Response Surface Methods including Gaussian processes for interpolation of solutions [1, 16, 24, 41].

The applications are presented in the following order: We start with the experimental optimization of a burner in combustion processes in Sect. 2. Bluff body flow and particularly the minimization of cylinder drag is the topic of Sect. 3. In Sect. 4, we describe how aerodynamic profiles can be designed using novel stochastic optimization concepts. The optimization of micromixers is shown in Sect. 5, microchannel design in Sect. 6, jet mixing in Sect. 7, and aircraft trailing vortex destruction in Sect. 8. Our observations are concluded in Sect. 9.

2 Multi-Objective Optimization in Combustion Processes

The optimization of the combustion process of a stationary gas turbine is a challenging real-world application with conflicting objectives. New governmental laws on emission taxes and global agreements on emission reduction such as the Kyoto resolution on greenhouse-gases (1997, 2001) demand expensive, highly thermodynamically efficient power plants with low emissions. On the other hand, the liberalization of the electric power market puts pressure on overall production costs. In recent years, the use of gas turbines among new power plants has significantly increased due to a number of appealing properties: Using natural gas instead of coal or oil leads to a cleaner combustion, while moderate installation and operating costs and a high thermodynamically efficiency reduce overall energy production costs. Moreover, using the exhaust heat for a steam turbine in a combined cycle is one way to increase power output and efficiency of the plant.

A central component in the design of a gas turbine is the design of the burners in the combustion chamber. The burners mix air and fuel and combust them continuously. This is different to Diesel engines, which combust in a cyclic manner. The design of a burner addresses two main objectives: First, the burner should mix air and fuel uniformly for low emissions, since the presence of areas of rich combustion results in increased NO_x emissions and a non-homogeneous temperature distribution may damage the turbine blades. Second, the burner should produce a stable combustion flame, avoiding undesired pulsations. Pulsations are due to thermo acoustic waves, which occur in particular for lean combustion when operating under part load condition. They reduce the lifetime of the turbine by fatigue and by destroying the film cooling along the blades surface. These two objectives are conflicting, thus motivating the requirement for a variety of designs as manifested on a Pareto front. The lack of viable analytical models and the limited information about the underlying physical processes involved makes combustion processes a suitable candidate for the optimization using stochastic optimization techniques [13].

Our contribution has been the application of multi-objective evolutionary optimization to a realistic industrial set-up [5]. Experimental setups present a number of challenges to any optimization technique including: availability only of pointwise information, experimental noise in the objective function, uncontrolled changing of environmental conditions and measurement failure. Based upen the SPEA [44], our evolutionary algorithm incorporates a number of new concepts, as dictated by the experiments, such as domination dependent lifetime, re-evaluation of solutions, and modifications in the update of the archive population.

We consider the optimization of a single burner in an atmospheric test-rig. Preheated air enters the test-rig from the plenum chamber and is mixed with fuel in the low-emission burner by swirl. The burner stabilizes the combustion flame in a predefined combustion area by a controlled vortex breakdown. The fuel is natural gas or oil and is injected through injection holes, which are uniformly distributed along the burner. A detailed description is given by Jansohn et al. [21]. Various investigations aimed to reduce pulsations and emissions of the burner by active and passive control mechanisms. We consider a passive control mechanism, choosing the fuel flow rates through the injection holes of the burner as design variables of the setup, due to the low modification cost for the gas turbine compared to an active control system. Eight continuous valves are used to control the fuel rates. Each valve controls the mass flow through a set of adjacent injection holes along the burner axis.

The Pareto front is constructed for the objectives of minimization of NO_x emissions and reduction of the pressure fluctuations of the flame, yielding reduced emissions and pulsation of the burner [5]. The results from this work have led to three patents for new burner designs [13, 14].

3 Cylinder Drag Minimization

A real coded genetic algorithm was implemented for the optimization of actuator parameters for cylinder drag minimization. We consider the two-dimensional and incompressible flow at $Re = 500$ past a circular cylinder, in combination with two types of idealized actuators that are allowed either to move steadily and tangentially to the cylinder surface ("belts"), or to steadily blow/suck with a zero net mass constraint. The genetic algorithm that we implemented has the property of identifying minima basins, rather than single optimum points. The knowledge of the shape of the minimum basin enables further insights in the system properties and provides a sensitivity analysis in a fully automated way. The drag minimization problem is formulated as an optimal regulation problem.

By means of the clustering property of the present genetic algorithm, a set of solutions producing drag reduction of up to 50% is identified. A thorough cluster analysis [27] revealed that the important parameters for the flow control are only the ones corresponding to actuators containing the separation point in the uncontrolled flow. At the same time all the other actuators could be sliding/blowing/sucking with random velocities or remain fixed. To verify this hypothesis another validation run was performed, this time maintaining active only the relevant actuators.

A comparison between the two types of actuators, based on the clustering property of the algorithm indicates that blowing/suction actuation parameters are associated with larger tolerances when compared to optimal parameters for the belt actuators. The possibility to use few strategically placed actuators in order to obtain a significant drag reduction was explored using the clustering diagnostics of this method. The optimal belt-actuator parameters obtained by optimizing the two-dimensional case have been employed in three dimensional simulations, by extending the actuators across the span of the cylinder surface. The three dimensional controlled flow exhibits a strong two-dimensional character near the cylinder surface, resulting in significant drag reduction [27].

The results obtained using two dimensional simulations are shown to be useful for three dimensions when the actuators are suitably extended on the third dimension of the flow [27]. This suggests that optimization in two dimensions followed by a validation of the results in three dimensions is a viable approach to the rapid design of realistic control devices.

4 Aerodynamic Profile Design

We consider the automated profile design for compressor blades of stationary gas turbines. The design is restricted to blades with subsonic flow. An optimization loop is implemented comprising an optimization algorithm, a profile generation tool and a computational fluid dynamics (CFD) analysis tool. The profile generator describes the profile by a set of Bezier splines whose control points are encoded into engineering design parameter like the profile length, the nose and trailing edge radius, and the curvature distribution [22, 42]. The flow analysis is performed with MISES [15], a quasi 3D computational fluid dynamics solver, which solves the Euler equation with an integral, viscous boundary layer formulation. It takes into account the change in the streamline thickness along the profile (quasi 3D). Our approach is to calculate various incidences in order to approximate the loss polar of the profile as given in Fig. 2. The loss polar specifies the behavior of the profile over the complete operating range. A disadvantage is the large number of flow calculations, which are needed to specify the polar as in the optimization of [22]. Furthermore, there is the problem of how many incidences should be computed and for which values.

In the following, we do not compute the complete loss polar and show that it is sufficient to compute 3 different incidences in order to assess a profile. The 3 calculations are performed for the design condition, i.e. $0°$ incidence and for one positive incidence $I_1 > 0°$ and one negative incidence $I_2 < 0°$. The key concept is to define I_1 and I_2 by a free multiplier θ as $I_1 = 1.0 \cdot \theta$ and $I_2 = -0.8 \cdot \theta$. This definition takes into account that the positive incidence I_1 is more critical for stall than I_2. The incidence multiplier θ is an additional design variable. The profile losses for the 3 incidences are summed to the first objective function f_1. For small values of θ, the losses are computed at small incidences. An optimization for small values of θ leads profiles which have minimal losses in the vicinity of the design condition, while for large values of θ, profiles are optimized for a large incidence range. Thus, θ is not only used as free design variable, but also as second objective function f_2. We minimize f_1 and maximize f_2 where the objective functions include penalties: $f_1 = \sum_{i=1}^{3} l_i + p_1 + p_2 + p_3 + p_4$ and $f_2 = \theta - p_1$, where l_i is the profile loss for the incidence i and p_1 to p_4 are 4 penalties, which are non-zero, if the corresponding constraint is violated. The first p_1 penalty regards convergence of the CFD solver. Penalty p_2 to p_4 address flow separation and mechanical stresses. The 15 free design variables are the parameters from the profile generator and the incidence multiplier θ. Two optimization runs are performed for a profile design at an inlet Mach number of 0.67, a desired flow turning of 12° and $\delta\beta = 0.1°$. In the first optimization, the two conflicting objectives f_1 and and f_2 are aggregated and a single objective algorithm is used. The second optimization run is a Pareto optimization for the two conflicting objectives. Most optimization algorithm are designed for a single objective function. Thus, for considering multiple objectives, the objectives have to be aggregated into a single figure of merit f_M, which is then optimized. Here, we restrict ourselves to minimization of the figure of merit and construct it as $f_M = f_1 - f_2$.

We compare the convergence properties of the CMA evolution strategy and the optimization algorithm including a Gaussian Process model, respectively. A separete Gaussian Process is constructed for the loss at each design incidence as well as for each constraint and the prediction of all models is aggregation in order to approximate the merit function. First, 100 solutions are computed randomly and then the model is used to search for promising solutions. The model is always trained with all currently evaluated solutions.

In Fig. 1, the merit function is plotted over the number of design evaluations N. The CMA-ES converges by constantly decreasing the merit function and $f_M = -1.6868$ is obtained as best function value after 1000 function evaluations. In the figure, the merit function for the algorithm using the model decreases by a large value as the model is firstly used at $N = 100$ evaluations. The initial 100 random solutions are already sufficient to approximate the merit function well. After $N = 300$ evaluations, the best function value

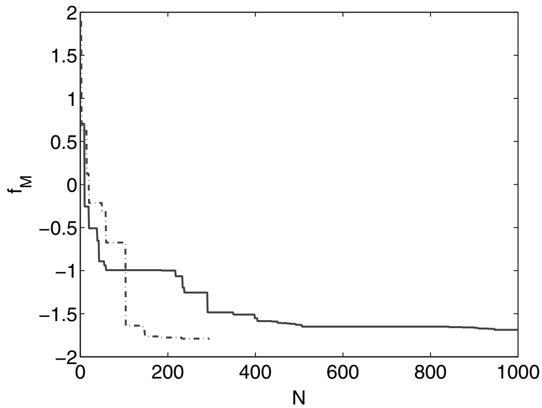

Fig. 1. The merit function versus the number of design evaluations for the CMA (−) and for the Gaussian process model (- - -)

is $f_M = -1.7892$. The plot shows the superior performance of the Gaussian process model compared with the CMA-ES.

We consider the optimization of the two objectives as a Pareto optimization problem. The first objective f_1 contains the losses, which is to be minimized. The second objective f_2 is the incidence multiplier and is to be maximized. The Pareto front underlines the conflict in optimizing the two objectives. For small incidence multipliers, the losses are low, since all 3 incidences are computed almost at the design point. For large incidence multipliers, the loss increases for two reasons. First, the flow is computed at larger incidences leading to higher losses and second, the profile losses are higher at the design condition, since the design has to be more robust for converging at the high incidences.

A multi-objective evolutionary algorithm with adaptive recombination and mutation operators is used for the Pareto optimization [7]. In total, 10.000 solutions are evaluated. Among all evaluated solutions, 5.461 solutions do not violate any constraints and generate a Pareto front of 283 solutions (Fig. 2). Two Pareto solutions are marked in the figure and their loss polar is given in Fig. 2. The minimal losses are at about 1.4 %. The attainable operating range is considered to be bounded by the double of the minimal losses [22]. Solution A contains the smaller incidence multiplier and the loss polar shows lower losses close to the design incidence than solution B, but comprises a smaller operating range. For solutions A and B, the operating range is about 14.4° and 15.5°, respectively. Both polars are characterized by a smooth and continuous increase of losses over the absolute incidence. This indicates a

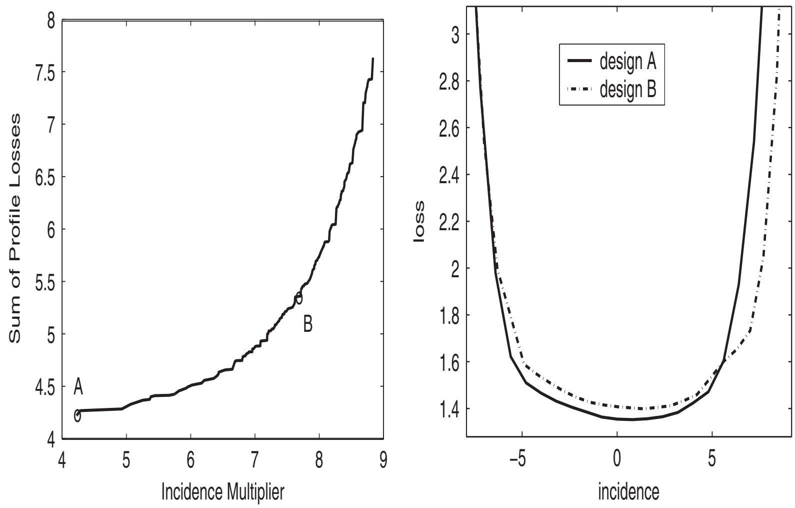

Fig. 2. Pareto front [*left*] for the profile optimization, and loss polar [*right*] for two selected Pareto solutions

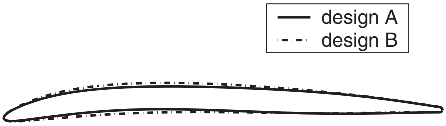

Fig. 3. Profile shape for the two selected Pareto solutions

soft stall behavior. Figure 3 contains the profile shape. Solution A shows the smaller nose radius as well as the smaller maximal thickness.

5 Micromixer

We studied mixing in a transverse-momentum micromixer for pharmaceutical applications. The mixer involves the parallel injection of two fluids which in an uncontrolled configuration do not mix due to the low Reynolds number of the flow. The flow configuration is shown in Fig. 4.

The control involves the use of side micropistons which should be activated so as to induce mixing. A straightforward trial and error experimentation with these actuations did not lead to any significant mixing. Extensive theoretical studies have identified suitable actuation parameters.

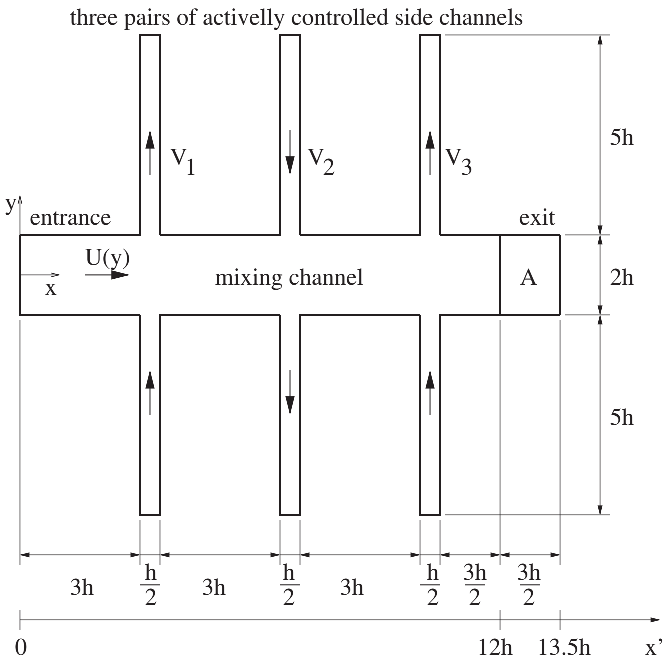

Fig. 4. Sketch of the flow configuration

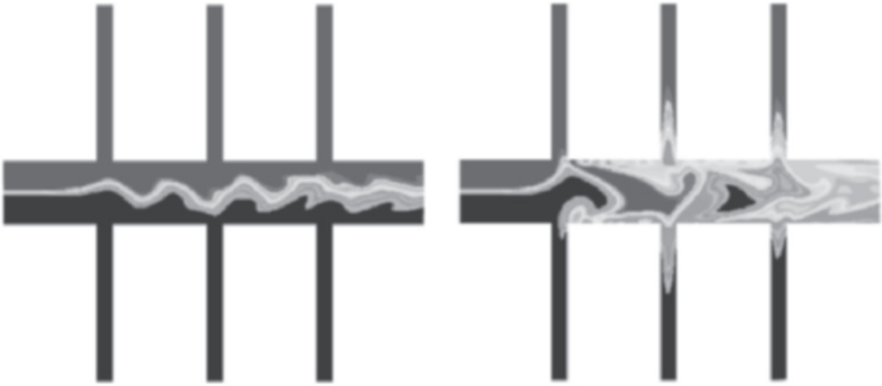

Fig. 5. Flow actuated by the initial frequencies $\boldsymbol{x} = (1/2, 1/2, 1/2)$ (*left*) and by the optimal frequencies $\boldsymbol{x} = (0.14, 0.32, 0.50)$ (*right*)

Our approach was to combine evolution strategies as optimization method with the simulation of the mixing behavior of the two fluids. The flow is modeled by the Navier-Stokes and convection-diffusion equations discretized using a second order finite volume technique and solved on a Cartesian grid using a standard computational fluid dynamics package [40]. The chosen optimization strategy was an evolution strategy with covariance matrix adaptation [17, 18, 19]. Optimization parameters are the frequencies of the movement of the micropistons and the objective is to increase the mixing of the two fluids which is estimated from the local variance of the concentration field.

Figure 5 shows two snapshots of the flow in the micromixer at time $t = 45$ for initial and optimal frequencies, respectively.

It was shown that the evolution algorithms can identify, in an automated fashion, effective actuations with mixing results that far exceeded those obtained by theoretical studies for the same configuration. In addition, we found that optimal frequencies for an increasing number of transverse channels are superposable despite the nonlinear nature of the mixing process [29, 30, 32].

6 Microchannel Flow

We apply both single and multiobjective EAs applied to a fluidic microchannel design problem [37, 38]. Bio-analytical applications require long thin channels for DNA sequencing by means of electrophoresis. In order to pack a channel of several meters in length onto a small square plate, curved geometries are required. However, curved channels introduce dispersion and, therefore, limit the separation efficiency of the system. The question is how to shape the contour of the channel in order to minimize dispersion. A detailed description of the problem as well as an optimization solution using gradient methods can be found in [28].

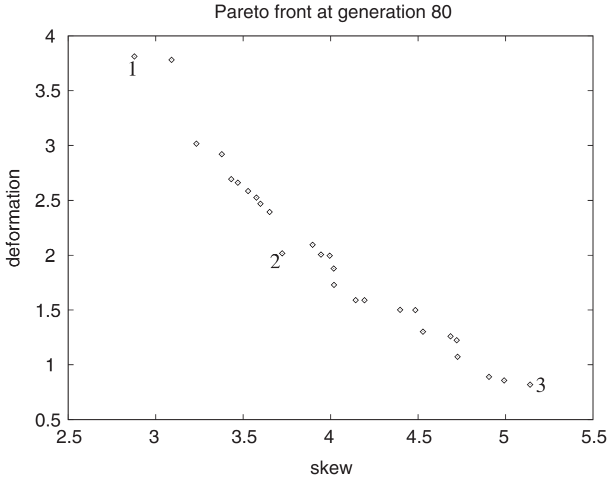

Fig. 6. Pareto-front of nondominated solutions after 80 generations

In a first study, we study a single-objective: The goal is to minimize the final skew of the flow inside the channel, i.e., it is required that the iso-values of the advected species be normal to the flow field when they exit the channel. The shape of the 90-degree turn is described by 11 parameters. The main result of our optimization using a $(1 + 1)$-ES with $1/5$ success rule is that we find a novel double-bump shape that has not been found before. Previous studies had shown single-dented designs always.

In a second study, we optimize two goals, the skew and the total deformation of the channel contour. The second goal is introduced to take into account manufacturing costs which increase when the channel is deformed strongly. The results from our two-objective minimization using the Strength Pareto Evolutionary Algorithm [44] are compared with the gradient-based optimization results by [28]. Figure 6 shows the Pareto-optimal trade-off front after 80 generations of the algorithm, and Fig. 7 shows the corresponding solutions, i.e., optimized shapes of the channel. From this front, we can decide for a solution with minimal skew at the expense of a higher deformation (point 1; represented in Fig. 7, left), some intermediate result (point 2), or with minimal deformation with the lowest skew possible (point 3, represented in Fig. 7, right).

Figure 8 shows two classes of optimized shapes obtained by [28] using gradient methods. Interestingly, the gradient technique offers two different designs, namely the single-dented (Fig. 8, left) and the double-dented shapes (Fig. 8, right) which we found with the evolution strategy also. Therefore, we obtain qualitatively similar results from both methods. Using the gradient method, the skew is reduced by one order of magnitude [28] which is comparable to the numbers obtained by evolutionary optimization. While trial and error procedures were used in the gradient methods to obtain various

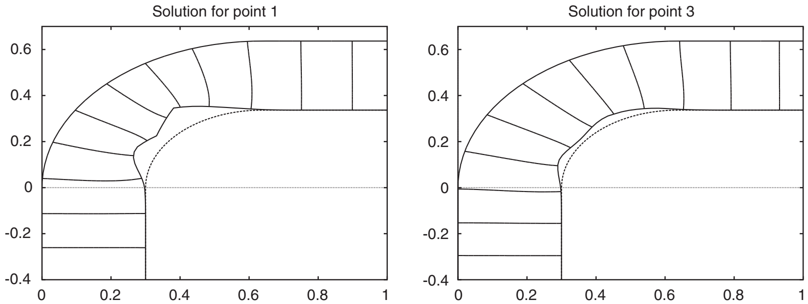

Fig. 7. Solution at point 1 (*left*) and at point 3 (*right*)

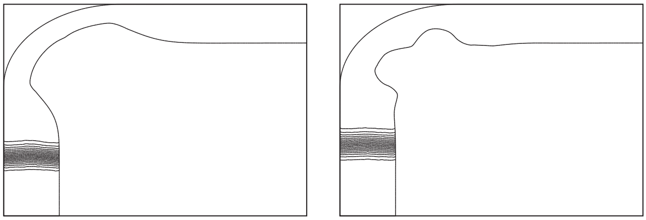

Fig. 8. First (left) and second (right) optimized shape using a gradient method [28]

solutions, ES provides us with a number of solutions and a Pareto front in an automated fashion. Unlike the gradient based methods which require an explicit formulation of the optimization problem at hand, the evolution strategy provides a straightforward optimization procedure.

7 Jet Mixing

Enhanced jet mixing has several technological applications with the goal of improving safety, efficiency, or reliability. One example relevant to military aircraft is signature suppression where the dispersion of the hot jet exhaust is aimed at. Another example in jet propulsion is to reduce the plume temperature on aerodynamic surfaces, such as the blown flap of a C-17 aircraft. In combustion processes, it is often important to enhance the turbulent mixing of the chemical components to make the combustion process more efficient with size and weight reductions possible, and to reduce the concentration of pollutants. The mixing rate of a jet can be significantly altered by applying a suitable excitation at the jet orifice. Since the external forcing interacts with the natural modes of the jet in a nonlinear way, it is difficult to estimate which kind of actuation is optimal to increase mixing.

Our work has focused on jet optimization using DNS and LES of an incompressible jet [30, 32]. We have studied helical and combined helical and axial forcing of a jet that maximizes mixing by combining an evolution strategy

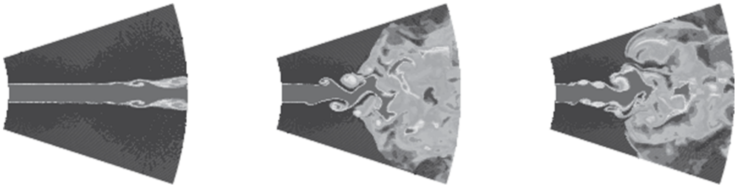

Fig. 9. Jet at Re = 1500. Left: No actuation. Middle: With maximum radial velocity for one-frequency excitation; Strouhal frequency $\mathrm{St}_h = 0.30$, helical amplitude $A_h = 0.08$. Right: With maximum radial velocity for two-frequency excitation; axial Strouhal frequency $\mathrm{St}_a = 0.72$, helical Strouhal frequency $\mathrm{St}_h = 0.29$, axial amplitude $A_a = 0.025$, helical amplitude $A_h = 0.075$

with direct numerical simulation of a round jet. Varying the actuation at the orifice, we searched for the forcing that maximizes various metrics related to enhanced jet mixing. For one- and two-frequency actuation, we have obtained optimum mixing results presented in Fig. 9.

Since DNS of high Re flows were becoming exceedingly expensive, we considered vortex filament methods as a model for high Re flows [9]. The three-dimensional evolution of a nominally axisymmetric jet subject to azimuthal and helical perturbation waves has been studied [25]. Circular filaments are uniformly initialized, with uniform circulation, and then slightly displaced to generate the perturbation. We performed evolutionary optimization, starting from values given by [26], that resulted in an improvement of the total length of the filaments (the mixing metric) of 42 % compared to the initial values.

The main finding from the jet mixing optimization using both vortex models and direct simulations was that the upper bound of the actuation amplitude appears to be of utmost relevance to mixing. A study of the physics of mixing has yet to reveal why this behavior emerges.

We have extended the implementation of an evolution strategy to the experimental laboratory at Stanford University [37]. We have used the original device developed by Parekh and Reynolds to study bifurcating jets. By automating the control process using LABVIEW™ and by coupling this process to our optimization algorithm we have obtained a 50% enhancement in the temperature mixing profile as compared to the classic experiments [33].

8 Aircraft Trailing Vortex Destruction

Trailing vortices are naturally shed by airplanes. They result in a strong downwash which extends for several miles behind the plane and poses a hazard to following aircraft, in particular at take-off and landing. Several previous studies propose to alleviate the hazard by introducing perturbations to trigger instabilities, and ultimately, break up the vortices [2, 12].

Recent studies [10, 35] have considered instabilities unique to several pairs of vortices which model aircraft wakes in landing configuration [39]. Some of

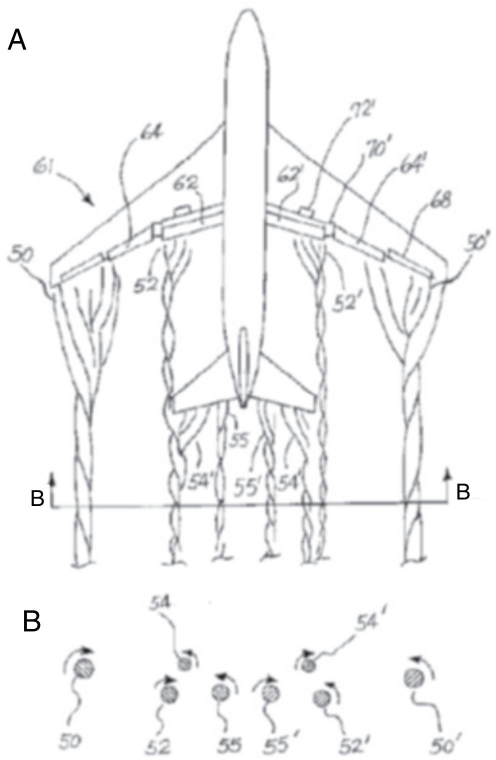

Fig. 10. Sketch of vortex system shed by an airplane (Courtesy of Crouch and Spalart [11]). B is a cross section of A as shown

these vortices quickly merge, but others persist for long times. At a distance of several spans behind a typical airplane, three persistent vortex pairs can generally be observed, originating at the tips of the wings, the outboard flaps, and the fuselage, see Fig. 10.

Crouch [10] has studied the linear stability of two pairs of co-rotating vortices (tip and outboard flap). He identified several instability modes depending on the angle, wavelength, and amplitudes of the perturbations that are imparted to each pair. Although the points of view adopted in recent studies [10, 11, 35] differ in several respects, in particular in the way the instability growth is measured, they have in common that they can provide us with a better understanding of the mechanisms by which the cooperative instabilities of several pairs can result in enhanced growth rates. Moreover, the configurations studied in these works are investigated with a view to implementing them in actual wing designs. One of the findings reported in [10] and [35] is the extreme sensitivity of the overall dynamics with respect to the initial state of the vortex pairs. In [10], the most effective transient growth was achieved when the outboard pair was not initially perturbed, while in [35] early reconnection was obtained for a particular value of the inboard vortices separation.

This motivates our attempt to perform a more systematic parameter search and identify the wake system which would produce the largest instability growth. Our goal is to revisit the above studies using viscous vortex methods and optimization with evolution strategies. Vortex methods are well adapted to wake simulation as they require the discretization of only the region of vorticity [9]. Note that the work of [35] is in part based on a vortex filament method. Using a $(1+1)$-evolution strategy, we optimize a total of seven parameters describing the perturbation of two pairs of co-rotating vortices, the tip and outboard vortices, and the geometry. The objective function to be maximized is the instability on the tip vortex.

We compare the results from the evolutionary optimization with parameters reported in [10] as leading to efficient transient growth. Some striking similarities can be noticed between these two sets of parameters. In particular, the ES has selected perturbations that are mostly located on the tip vortex, confirming the observation from linear stability analysis in [10] of efficient transient growth when the outboard flap vortex is unperturbed. The wavelengths of the perturbations are also close to the ones given in [10]. The case of four pairs of vortices is also considered and leads to a larger distortion of the tip vortex [37].

9 Summary and Conclusions

Biologically inspired stochastic search algorithms were applied to a variety of engineering problems ranging from aerodynamics and turbomachinery to microtechnology. In summary, the results show that these optimization methods are highly suitable for optimization in applications that are characterized by noise, multimodality, and no availability of gradient information.

References

[1] El-Beltagy, M.A., Keane, A.J., "Evolutionary optimization for computationally expensive problems using gaussian processes," Hamid Arabnia, editor, *Proc. Int. Conf. on Artificial Intelligence IC-AI'2001, Las Vegas*, pp. 708–714. CSREA Press, 2001.

[2] Bilanin, A.J., Widnall, S.E., "Aircraft wake dissipation by sinusoidal instability and vortex breakdown," AIAA 73–107, 1973.

[3] Bischof, C., Griewank, A., "Tools for the automatic differentiation of computer programs," *ICIAM/GAMM 95: Issue 1. Numerical Analysis, Scientific Computing, Computer Science*, pp. 267–272, 1996.

[4] Booker, A.J.J., Dennis Jr, J., Frank, P.D., Serafini, D., Torczon, V., Trosset, M.W., "A rigorous framework for optimization of expensive functions by surrogates," *ICASE report No. 98–47*, NASA Langley Research Center, Hampton, VA, 1998.

[5] Büche, D., Stoll, P., Dornberger, R., Koumoutsakos, P. "Multi-objective Evolutionary Algorithm for the Optimization of Noisy Combustion Problems," *IEEE Transactions on Systems, Man, and Cybernetics, Part C*, Vol. 32, No. 4, pp. 460–473, 2003.

[6] Büche, D., Guidati, G., Stoll, P., Koumoutsakos, P. "Self-organizing maps for pareto optimization of airfoils," *Seventh International Conference on Parallel Problem Solving from Nature (PPSN VII)*, Granada, Spain. Springer-Verlag, 2002.

[7] Büche, D., Milano, M., Koumoutsakos, P., "Self-Organizing Maps for Multi-Objective Optimization," *Workshop Proceedings of the Genetic and Evolutionary Computation Conference (GECCO)*, Morgan Kaufmann Publishers, San Francisco, CA, pp. 152–155, 2002.

[8] Büche, D., Milano, M., Koumoutsakos, P., "Growing Self-Organizing Maps for Multi-Objective Optimization," Evolutionary Computation Journal, 2002 (submitted).

[9] Cottet, G.H., Koumoutsakos P.D., "Vortex Methods – Theory and Practice," Cambridge University Press, 2000.

[10] Crouch, J.D., "Instability and transient growth for two trailing-vortex pairs," *Journal of Fluid Mechanics*, No. 350, pp. 311–330, 1997.

[11] Crouch, J.D., Spalart, P.R., "Active system for early destruction of trailing vortices," *US Patent 6082679*, issued July 4, 2000.

[12] Crow, S.C., Bate, E.R., "Lifespan of trailing vortices in a turbulent atmosphere," *Journal of Aircraft*, No. 13, pp. 476–482, 1997.

[13] Dornberger, R., Stoll, P., Paschereit, Ch.O., Schuermans, B., Bueche, D., Koumoutsakos, P., "Numerisch experimentelle Optimierung des Mischungsprofils für die Kontrolle verbrennungsgetriebener Schwingungen und Emissionen", *German Patent No. 101 04 151.9*, ALSTOM Power (Schweiz) AG, 2001.

[14] Dornberger, R., Stoll, P., Paschereit, Ch.O., Schuermans, B., Bueche, D., Koumoutsakos, P., "Redundanzfreier Ansteuerungsmechanismus der Ventile zur Beeinflussung des Mischungsprofiles für die Kontrolle verbrennungsgetriebener Schwingungen, *German Patent No. 101 04 150.0*, ALSTOM Power (Schweiz) AG, 2001.

[15] Drela, M., Youngren, H., "A User's Guide to MISES 2.53," MIT, 1998.

[16] Gibbs, M.N., MacKay, D.J.C., "Manual for tpros, v2.0," 1997.

[17] Hansen, N., Ostermeier, A., "Convergence Properties of Evolution Strategies with the Derandomized Covariance Matrix Adaptation: The $(\mu/\mu_I, \lambda)$-CMA-ES," *Proceedings of the 5th European Congress on Intelligent Techniques and Soft Computing (EUFIT'97)*, pp. 650–654, 1997.

[18] Hansen, N., "Verallgemeinerte individuelle Schrittweitenregelung in der Evolutionsstrategie: Eine Untersuchung zur entstochastisierten, koordinatensystemunabhängigen Adaptation der Mutationsverteilung," Mensch & Buch Verlag, Berlin, 1998.

[19] Hansen, N., Ostermeier, A., "Completely Derandomized Self-Adaptation in Evolution Strategies," *Evolutionary Computation*, Vol. 9, No. 2, pp. 159–195, 2001.

[20] Hansen, N., Müller, S.D., Koumoutsakos, P., "Reducing the Time Complexity of the Derandomized Evolution Strategy with Covariance Matrix Adaptation (CMA-ES)," *Evolutionary Computation Journal*, MIT Press, Vol. 11, No. 1, 2003.

[21] Jansohn, P., Ruck, T., Steinbach, C., Knöpfel, H.-P., T. Sattelmayer, T., "Development of the Advanced EV (AEV) Burner for the ABB GTX 100 Gas Turbine", *ASME Turbo Asia 97*, Singapore, 1997.

[22] Köller, U., Mönig, R., Küsters, B., Schreiber, H.-A., "Development of Advanced Compressor Airfoils for Heavy-Duty Gas Turbines, Part I: Design and Optimization," *ASME Journal of Turbomachinery*, Vol. 122, No. 3, pp. 397–405, 1999.

[23] Kohonen, T., "Self-organizing maps." *Springer series in information sciences*, 3rd edition, 2001.

[24] "Gaussian processes - a replacement for supervised neural networks," *Lecture notes for a tutorial at NIPS*, 1997.

[25] Martin, J.E., Meiburg, E., "Numerical investigation of three-dimensionally evolving jets under helical perturbations," *Journal of Fluid Mechanics,* Vol. 243, pp. 457–487, 1992.

[26] Martin, J.E., *Personal communication,* 2001.

[27] Milano, M., Koumoutsakos, P., "A Clustering Genetic Algorithm for Cylinder Drag Optimization," *Journal of Computational Physics*, Vol. 175, pp. 79–107, 2002.

[28] Mohammadi, B., Molho, J.I., Santiago, J.A., *Design of Minimal Dispersion Fluidic Channels in a CAD-Free Framework*, Center for Turbulence Research Annual Research Briefs, 2000.

[29] Müller, S.D., Sbalzarini, I.F., Walther, J.H., Koumoutsakos, P., "Evolution Strategies for the Optimization of Microdevices," *Proceedings of the Congress on Evolutionary Computation (CEC 2001)*, pp. 302–309, 2001.

[30] Müller, S.D., Koumoutsakos, P., "Mixing Optimization with Evolution Strategy," *Evolutionary Methods for Design, Optimization and Control with Applications to Industrial Problems – Proceedings of the EUROGEN 2001 Conference*, pp. 448-452, September 2001.

[31] Müller, S.D., Hansen, N., Koumoutsakos, P., "Improving the Serial and Parallel Performance of the CMA-Evolution Strategy with Large Populations," *Proceedings of the Seventh International Conference on Parallel Problem Solving from Nature (PPSN VII)*, Lecture Notes in Computer Science, Vol. 2439, Springer, Berlin, pp. 422–431, 2002.

[32] Müller, S.D., Koumoutsakos, P., "Control of Micromixers, Jets, and Turbine Cooling Using Evolution Strategies," *Manipulation and Control of Transverse Jets*, International Centre for Mechanical Sciences (CISM), Udine, Italy, 2002.

[33] Parekh, D., Leonard, A., Reynolds, W.C., "Bifurcating of Round air jets by dual-mode acoustic excitation," AIAA 87–0164, 1987.

[34] Rennich, S.C., "Accelerated destruction of aircraft wake vortices," *Ph.D. thesis*, Stanford University, 1997.

[35] Rennich, S.C., Lele, S.K., "A method for accelerating the destruction of aircraft wake vortices." AIAA 98–0667, 1998.

[36] Reuther, J.J., Alonso, J.J., Jameson, A., Rimlinger, M.J., Saunders, D., "Constrained multipoint aerodynamic shape optimization using an adjoint

formulation and parallel computers, Part I and II," AIAA Journal of Aircraft, Vol. 36, No. 1, pp. 51–71, 1999.

[37] Sbalzarini, I.F., Müller, S.D., Koumoutsakos, P., Cottet, G.-H., "Evolution Strategies for Computational and Experimental Fluid Dynamics Applications," *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO)*, Spector, L., Goodman, A., Wu, A., Langdon, W.B., Voigt, H.-M., Gen, M., Sen, S., Dorigo, M., Pezeshk, S., Garzon, M., Burke, E. (eds.), Morgan Kaufmann Publishers, San Francisco, CA, pp. 1064–1071, 2001.

[38] Sbalzarini, I.F., Müller, S.D., Koumoutsakos, "Optimization Using Multiobjective Evolution Strategies," *Proceedings of the First International Conference on Evolutionary Multi-Objective Optimization (EMO'01)*, Lecture Notes in Computer Science, Vol. 1993, Springer, Berlin, 2001.

[39] Spalart, P.R., "Airplane trailing vortices," Annual Review of Fluid Mechanics, No. 30, pp. 107–138, 1998.

[40] *Star-CD User Manual,* Computational Dynamics Ltd., London, 1997.

[41] Torczon, V., Trosset, M.W., "Using approximations to accelerate engineering design optimization," *ICASE Report No. 98–33*, Technical report, NASA Langley Research Center Hampton, VA, 23681-2199, 1998.

[42] Trigg, M.A., Tubby, G.R., Sheard, A.G., "Automatic Genetic Optimization Approach to Two-Dimensional Blade Profile Design for Steam Turbines," *ASME Journal of Turbomachinery*, Vol. 121, No. 1, pp. 11–17, 1999.

[43] Volpert, M., Mezic, I., Meinhart, C.D., Dahleh, M., "Modeling and Analysis of Mixing in an Actively Controlled Micromixer," *Unpublished report, University of Santa Barbara, CA*, 2000.

[44] Zitzler, E., Thiele, L., "Multiobjective Evolutionary Algorithms: A Comparative Case Study and the Strength Pareto Approach," *IEEE Transactions on Evolutionary Computation*, Vol. 3, No. 4, pp. 257–271, 1999.