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AUTOMATED DESIGN OPTIMIZATION OF COMPRESSOR BLADES FOR STATIONARY, LARGE-SCALE TURBOMACHINERY

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ABSTRACT

An automated, multi-disciplinary optimization procedure for sub-sonic gas turbine compressor blades is presented. Evolutionary optimization algorithms are coupled with existing tools for geometry generation, mechanical integrity analysis and Q3D flow analysis for design and off-design conditions. Aerodynamic and mechanical objectives and constraints are formulated based on the standard design criteria. The feasibility of the approach is tested by automatically designing different rotor blades for the subsonic compressor region. First results are promising. All rotor blades show similar profile shapes, which underlines the robustness of the optimization procedure. The blades are characterized by a pronounced front loading which leads to a large (predicted) operating range. A special focus in this paper is on a 3D-blade parameterization, which by default leads to smooth blades, and on the assessment of the off-design behavior. The considered optimization algorithm shows a fast and robust convergence even from randomly initialized blades.

INTRODUCTION

A long term effort at the Alstom Power Technology Center is the development of automated optimization procedures for different areas. This paper presents an application in the field of compressor blade design. The design process of multi-stage axial compressors, as used in large-scale stationary gas turbines or aircraft engines, consists of a sequence of design steps. The first step is usually a mean-line calculation, which considers only the mid-span profile sections of the compressor. At this point, the pressure increase per stage is prescribed. Then, a meridional (S2) through-flow code (e.g. a streamline curvature method) is used to compute a radial distribution of the main aerodynamic properties of each row. This calculation is performed on a number of distinct streamlines. Their position is adapted in an iterative procedure until the radial equilibrium equation is fulfilled.

The first two steps depend mainly on the experience of the design engineers and define most of the compressor dimensions and properties, especially the aero-/thermodynamic boundary conditions up- and downstream of each blade row. At this stage only a rough model of the blade shape exists, i.e. the metal and stagger angles and an estimated profile thickness.

The actual blade design is performed on the basis of 2D cuts (profiles), which are stacked to a 3D blade. These profiles are designed and analyzed on conical S1 surfaces. The latter are defined by the meridional streamlines, which result from the preceding S2 calculation. The Q3D aerodynamic analysis takes into account the streamline thickness and radius variation. This simplification omits 3D flow effects. The validation of a Q3D optimization of Chung et al. (2001) for a transonic compressor blade show a reduced performance gain, when comparing the Q3D results with a 3D CFD computation. Thus, Q3D optimization should be limited to subsonic flows, due to the 3-dimensional shock structure in transonic blade rows.

The objective of the profile design is to find for each streamline a shape, which fulfills the aerodynamic boundary conditions of the S2 calculation, especially in terms of the flow turning. In addition low aerodynamic losses at the design point and off-design conditions are desired. Finally, the resulting 3D blade must sustain aerodynamic and centrifugal forces, avoid critical resonance frequencies, and respect manufactural constraints such as smoothness.

Complete 3D CFD analysis and 3D mechanical integrity calculations are often performed after the design process for the design assessment, since they are, even on today's computer clusters, computationally expensive. Hence, a large part of the engineer's work is concerned with the design of 2D profile sections. This task comprises several (partly conflicting) objectives and constraints and is solved by a designer in a time-consuming iterative process. Here, an automated optimization procedure leads to the most significant labor time reduction for the designer.

Aerodynamic design optimization is distinguished into inverse and direct design. In inverse design, a pressure distribution is prescribed and the according profile shape is searched by an iterative modification of the blade shape. The computational cost is proportional to a few flow analysis and thus comparably cheap. However, an iteration of the prescribed pressure distribution may be necessary to obtain an acceptable profile. This approach significantly relies on the experience of the designer, who needs to specify a pressure distribution, which fulfills the various aerodynamic design aspects in terms of flow turning, boundary layer properties and losses, and which performs also well for off-design condition. According to Reuther et al. (1999), varying the pressure distribution cannot guaranty to improve a design and might even be unrealizable, especially for 3D configurations. A shortcoming of inverse design is the open question of how to integrate geometrical and mechanical constraints (Shahpar 2000).

Direct design considers the shape optimization for secondary aerodynamic properties like aerodynamic losses and the computational cost are a multiple of a single flow calculation. About 20 years ago, Sanger (1983) presented a pioneer work in automated optimization of a high-subsonic compressor profile by numerical flow solvers. This optimization contained already all necessary ingredients for a successful profile optimization: a 2D potential flow solver with an integral formulation of a compressible boundary layer was used in conjunction with a gradient-based constraint optimization approach (CONMIN) for minimizing constraints on the main aerodynamic properties. The optimization results were obtained in 50 min.

In direct design, off-design conditions can be included by performing CFD calculations for various operation conditions (Egartner and Schulz 1998), (Köller et al. 2000), (Huyse and Lewis 2001). In addition, geometrical constraints can be included in the parameterization of the blade and mechanical constrains can be considered by additional solvers.

Optimization algorithms for direct design are mainly gradient-based methods and stochastic algorithms. Gradient-based methods rely on derivative information of all objectives and all constraints for determining the search direction of the optimization. Derivatives can be obtained by adjoint formations (Reuther et al. 1999), but codes are rarely available. In future, more derivative information might be available due to software tools like automatic differentiation (Bischof and Griewank 1996) or the complex-step method (Martins et al. 2000). In automated differentiation, the derivative is generated by a software library, which differentiates the code. This method is usually much more resource intensive than the adjoint method, but the derivative are obtained in an automated fashion. The complex-step method replaces all real numbers in the code by complex numbers and thus computing a single sensitivity is about twice as expensive as computing the original solver. An alternative is the computation of gradients by finite differences. This is not recommended, since finite differences are strongly affected by noisy solvers and the step sizes for the finite differences (Booker et al. 1998).

Genetic Algorithms and Evolution Strategies (Bäck and Schwefel 1993) are representatives of the class of stochastic optimization algorithms. These methods are considered as robust optimization algorithms, which are able to cope with noisy and multimodal functions, but are also computationally expensive in terms of the necessary number of flow analysis required for convergence. However, recent studies of Naujoks et al. (2000) show that advanced Evolution Strategies are able to optimize complex flow problems with limited resources by using advanced algorithms, which adapt the evolutionary search to the local topology of the optimization problem.

Response surface techniques are computationally cheap surrogates of the flow analysis and could also be used to accelerate the convergence. They have been successfully applied, e.g. to a helicopter blade optimization (Booker et al. 1998) and are especially interesting for very expensive solvers, since the necessary computational effort to build and analyze the response surface is usually smaller than the expense for one flow analysis. This seems a promising area for future research.

In the next sections, a direct design approach is presented for optimizing compressor blades. The approach comprises a stochastic optimization algorithm, in particular the Covariance Matrix Adaptation of Hansen and Ostermeier (2001), a blade parameterization for designing 2D profiles and 3D blades, and numerical tools for aerodynamic and mechanical integrity analysis. The aerodynamic analysis is performed by a quasi-3D CFD code on several radial blade sections. The mechanical integrity analysis estimates the maximum stresses and eigenfrequencies of the 3D blade. The design approach is tested on four sub-sonic rotor rows of a current Alstom compressor and the results are discussed.

DESIGN AND ANALYSIS TOOLS

The optimization process concerns the automated design of a single sub-sonic compressor row. The aerodynamic boundary conditions of a row are fully specified as the result of the S2 calculation in the meridional plane. In order to define and analyze blade shapes three steps are needed, namely (1) a profile/blade generator with an appropriate parameterization, (2) a mechani-

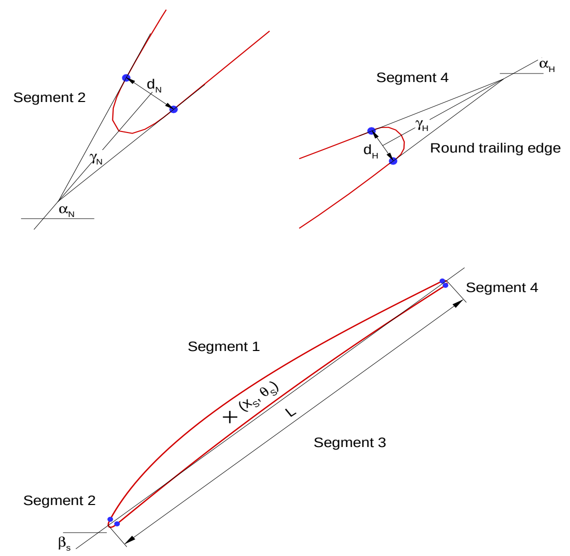


Figure 1. Parameterization of a compressor profile

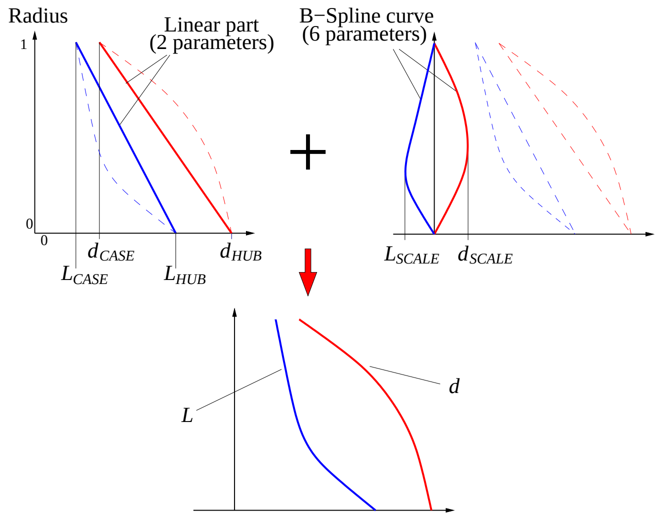


Figure 2. Definition of Radial shape functions

cal integrity analysis, and (3) a flow analysis. For this, existing in-house tools are used. These steps are described in this section.

Parameterization of Compressor Profiles and 3D Blades

A blade is described by a number of profile sections on the same conical surfaces, which are also used for the Q3D calculations. A profile section is constructed by four Bezier curves (suction surface, leading edge, pressure surface, and trailing edge). 6 control points define each segment, which leads to a total number of 48 parameters. 16 parameters are determined by enforcing C^2-continuity between the segments. The remaining 32 variables, i.e. Bezier point coordinates, are not modified directly but translated into engineering parameters, such as metal and wedge angles, profile length, etc. (see Fig. 1). Some of these are set to default values which leads to a final number of 19 parameters per profile section.

The transformation of the spline parameters to engineering parameters simplifies the comparison and illustration of different profile parameter sets. The profile sections can be described independently from each other. However, the radial distribution of the engineering parameters and some derived quantities, such the relative thickness, should be analyzed. A blade is considered acceptable if these quantities show a continuous distribution along the blade span without turning points. For the automated optimization an approach was chosen, which leads by definition to such a "smooth" blade. This was achieved by using a radial shape function for the variation of all engineering parameters along the blade span, which (1) has no turning points and (2) is defined by a number of parameters. These parameters are the actual design variables and considered by the optimization strategy. This approach ensures that a large variety of shapes can be realized while obeying the blade smoothness.

The radial shape function consists of two parts, namely (1) a linear variation between the hub and the casing streamlines, and (2) an additional B-spline curve which allows for variations from this basic shape (see Fig. 2). The linear part implies two free design variables, i.e. the value of the section parameter (e.g. L) at hub and casing, L_{HUB} and L_{CASE}, respectively. As shown in Fig. 3, the shape of the B-spline curve is defined by 5 parameters L_{SF0}-L_{SF4}. These parameters define the location of 3 B-spline points, which form the control polygon together with a start and end point. The convex hull property of B-splines implies that a convex control polygon leads to a convex curve. In order to avoid turning points in the curve, it is therefore necessary to ensure a convex polygon. This is achieved by a triangular coordinate system. The admissible range of L_{SF0}-L_{SF4} is between 0 and 1. An additional parameter L_{SCALE} determines, how strongly the radial shape function deviates from a straight line. Note that L_{SCALE} = 0.0 leads to a linear variation along the span.

For the first optimizations the parameters $*_{SF0}$-$*_{SF4}$ were fixed to 0.5 and only the $*_{HUB}$, $*_{CASE}$ and $*_{SCALE}$ variables for each engineering parameter $*$ were used as design variables. The B-spline curve (see Fig. 3) reduces then to a symmetric bump, similar to a parabola. Once the radial shape functions are defined by a set of design variables, the engineering parameters for each profile are obtained by computing the value of the radial shape functions at the radius corresponding to the streamline of interest. This procedure is outlined in Fig. 4. The streamlines are labeled SL1-SL9 from hub to casing.

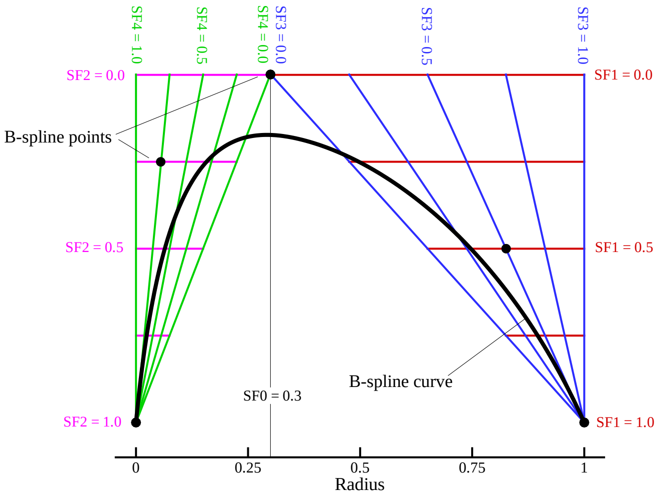


Figure 3. Definition of B-Spline curve

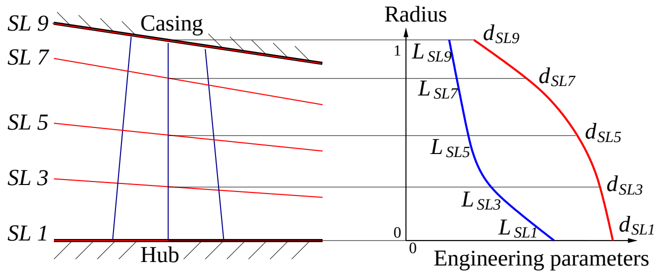


Figure 4. Link between radial shape function and engineering parameter

Mechanical Integrity Analysis

The mechanical integrity (MI) analysis assesses whether (1) a given blade shape can sustain the aerodynamic and centrifugal forces and (2) the eigenfrequencies do not match critical excitation frequencies. The results are translated into a number of safety factors. The sum of all negative factors (violated MI criteria) is called MI indicator. In order to demonstrate the principle of combined aerodynamical and mechanical blade optimization, a simplified beam model was used which runs only a few seconds on a modern workstation.

Q3D Flow Analysis

The flow analysis is performed for each profile section separately. This blade-to-blade (S1) computation is performed using the Mises (Drela and Youngren 1995) code. Mises solves an Euler equation for a single blade row and a single flow path, assuming periodical boundary conditions between two adjacent blades in a row. Viscous effects are modeled by a fully coupled

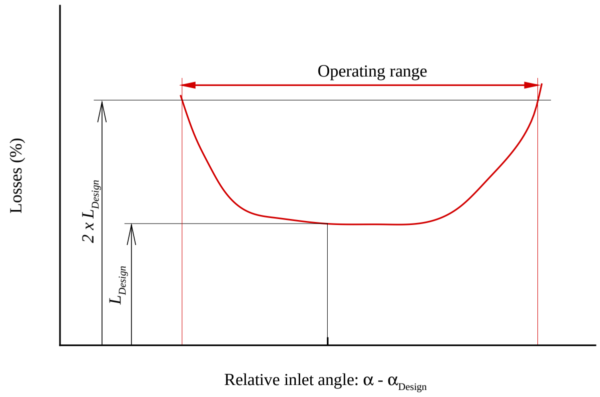


Figure 5. Definition of operating range

integral boundary layer formulation. The streamtube contraction as well as radius variations along the compressor axis are considered (Q3D).

A profile is assessed by performing the flow analysis for different inlet flow angles while keeping the inlet Mach number constant. Fig. 5 shows the aerodynamic losses as a function of inlet angle. The loss polar has typically a flat region around the design inlet angle and a sharp increase of losses for larger deviation from this design point. The operation range is defined by the range of incidence angles with losses below twice the losses at design incidence. Usually the operating range decreases with inlet Mach number.

OPTIMIZATION PROCESS

Optimization Algorithm

One prerequisite for a successful automated optimization of real world problems is the right choice of the optimization algorithm. Usually, for the various optimization problems, the different efficient optimization algorithms exist. Most of Alstom's design problems are evaluated with solvers, for which no explicit derivative information is available and noise is found in the objective function. Thus, gradient-based techniques are difficult to use and stochastic optimization algorithms are considered. A special focus is on Evolution Strategies (Rechenberg 1973). Evolution Strategies (ESs) imitate the natural evolution and contain operators such as mutation, recombination and fitness based selection.

First applications of evolution strategies to the optimization of aerodynamic devices were performed at the Technical University of Berlin, Germany in 1964 (Rechenberg 1973). Modern ES exploit information from the optimization history in order to adapt the mutation distribution to the local function topology and to increase the convergence speed. This allows converging toward the optimum with any precision. Especially the Covariance Matrix Adaptation (CMA) of Hansen and Ostermeier (2001) is a promising candidate for real-world applications. Naujoks et al. (2000) show the performance benefit of CMA compared to other ES for the optimization of aerodynamic profiles and show that the optimization can even be started from random initial profiles (as it is also done in this paper). This is a key advantage to gradient based methods, which rely on start solutions in the vicinity of the optimum.

Optimization Loop

An optimization loop consists of two main blocks, namely (1) the optimization algorithm and (2) the solver. The optimization algorithm is the CMA. It generates a certain number of individuals. An individual represents one blade design, given by a set of design variables. The design variables of the initial individuals are generated randomly, i.e. no initial solution is necessary. The individuals are passed to the solver. The solver implements the design tools and computes a fitness value for each individual. A fitness value is obtained by aggregating all objectives and all constraints to a single figure of merit. Then, the optimization algorithm selects the most promising solutions based on the fitness value, recombines and mutates these solutions in order to obtain new individuals, and the optimization loop restarts.

Fig. 6 shows the sequence of the different design tools in the solver. In the first step, the section parameters are computed from the design variables, using the radial shape functions. Then, the profile generator computes profile sections on all streamlines. In the next step, the MI indicator is computed. If the latter is below a user-defined limit, the Q3D flow analysis is performed, otherwise it is skipped and a fitness function is computed as a sum of the MI indicator and a penalty. The Q3D flow is computed on 3 streamlines at the hub, blade mid-span and tip. The flow analysis is the computationally most expensive part, especially if several incidence angles have to be calculated. Finally all objectives and penalties are aggregated into the fitness function (see below).

Design Variables

For the optimizations presented below, the number of possible design variables is reduced. As mentioned before, the 5 shape factors $*_{SF0}$-$*_{SF4}$ which describe the B-spline part of the radial shape function were set to 0.5. Hence, a complete blade is defined by 19 engineering parameters times the 3 parameters which describe the radial variation, i.e. $*_{HUB}$, $*_{CASE}$, $*_{SCALE}$. Some of the resulting 57 design variables were fixed, e.g. the location of the center of gravity or the profile length, for which good estimates are available already from the S2 calculation. Eventually, 27 design variables describe the geometry of a blade. As explained above, one of the criteria for a good compressor blade is the tolerance against off-design conditions. Since a computation of the complete loss polar is very time-consuming a simplified approach was developed. One additional design variable is introduced for each profile section, namely an inlet angle variation $\Delta\alpha$. The flow analysis is then performed at the design inlet angle α_{Design} and at $\alpha_{Design} \pm \Delta\alpha$.

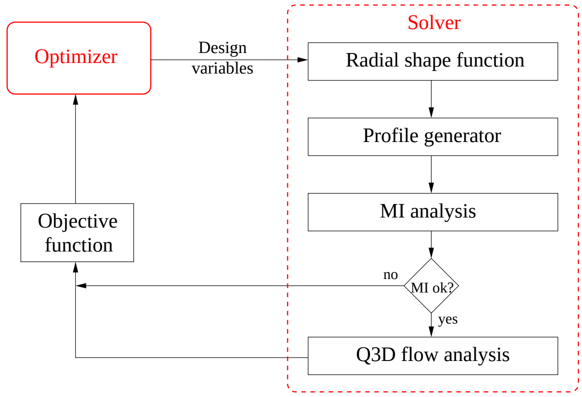


Figure 6. Illustration of optimization loop

Fitness Function

The fitness function f aggregates all objectives and constraints of the aerodynamical analysis and the MI indicator M_I by a weighted sum into a single figure of merit. The fitness is used by the optimizer to assess the quality of a design.

The main objective is to minimize the sum of aerodynamic losses for the design inlet flow angle $L_{\alpha_{Design}}$ and the two off-design angles $L_{\alpha_{Design}-\Delta\alpha}$ and $L_{\alpha_{Design}+\Delta\alpha}$ By subtracting the inlet angle variation $\Delta\alpha$, individuals with large $\Delta\alpha$ are favored and, as a consequence, the operating range is maximized, although it is not directly computed. Hence, the inlet angle variation is used as design variable and as objective (see Büche et al. (2002)).

Aerodynamical constraints are set on the deviation of the turning angle $\Delta\beta$ from the desired value. Furthermore, the maximum Mach number Ma and a characteristic boundary-layer quantity (in this case the non-dimensional shape factor H_{12}) at the suction side trailing edge are constrained. These quantities are limited in order to avoid transonic effects and flow separation, respectively. An additional penalty is added for poorly converged Mises computations. All aerodynamical quantities are summed up over the n distinct streamlines:

$$\begin{aligned} f = \sum_{i=1}^{n} \Big\{ & a_1 \left(L_{\alpha_{Design}} + L_{\alpha_{Design}-\Delta\alpha} + L_{\alpha_{Design}+\Delta\alpha}\right) \\ & - a_2\ \Delta\alpha \\ & + a_3 \max(0, |\Delta\beta| - \Delta\beta^{limit}) \\ & + a_4 \max(0, H_{12} - H_{12}^{limit}) \\ & + a_5 \max(0, Ma_{max} - Ma_{max}^{limit}) \Big\} \\ & + a_6\ M_I \end{aligned} \quad (1)$$

The weighting factors a_1 to a_6 have to be prescribed by the designer. This is done by choosing a tolerable margin for each component (e.g. 10% more losses or 0.05 higher peak Mach number). Then the weights are determined such that the different components of the objective function have similar magnitude in case the tolerable margin is reached. Clearly, the choice of the weighting is to some extent arbitrary and influences the final result. However, it also gives the designer a means of guiding the optimization in the desired direction.

OPTIMIZATION RESULTS

The automated optimization procedure was employed to the design of four rotor blades of a gas turbine compressor (Case A-D). A typical optimization run required approximately 4.000 computed designs. This corresponds to 1/2 day on a four-processor Linux cluster. The resulting Mach number distributions for the hub, midspan and tip profile sections are displayed in Fig. 7, 8 and 9. Compared to the conventional controlled-diffusion airfoils (CDA), the optimized profiles are more front-loaded. A comparison with optimizations for single incidence showed that this front-loading is driven by the demand for an increased operating range. It is interesting to note that the general trends for the Mach number distribution are similar for the different rows. This indicates the optimization procedure does not converge towards completely different optima.

Figure 10 compares the loss polar for the manually designed mid-span profile and the optimized profile for Case A. The operating range is indeed increased by 15% on both sides of the polar with the loss at design point being equal. The blades which result from the automated optimization procedure show a pronounced front loading when compared to the standard controlled diffusion airfoils (CDA). The latter exhibit a moderate acceleration of the flow on the suction side up to 10-20% chord. This allows in principle for a laminar flow up to the point where the flow deceleration starts, which in turn can lead to a reduction of viscous losses. However, for stationary turbomachinery compressors the high inlet Reynolds number $(1-2\ 10^6)$ and turbulence level (4%) likely effect a by-pass transition very close to the leading edge. Then there is no advantage of a moderate flow acceleration anymore. In contrary, it is better to have a strong acceleration and deceleration close to the leading edge where the turbulent boundary layer is thin and to reduce the deceleration towards the trailing edge where the boundary layer is thicker.

Airfoil shapes which are similar to the obtained designs were investigated numerically and experimentally by Köller et al. (2000) and Küsters et al. (2000). The profiles were also highly front-loaded and showed an early transition. The operating range was increased by 38 % compared to a conventional design, however, with the inlet Mach number being lower.

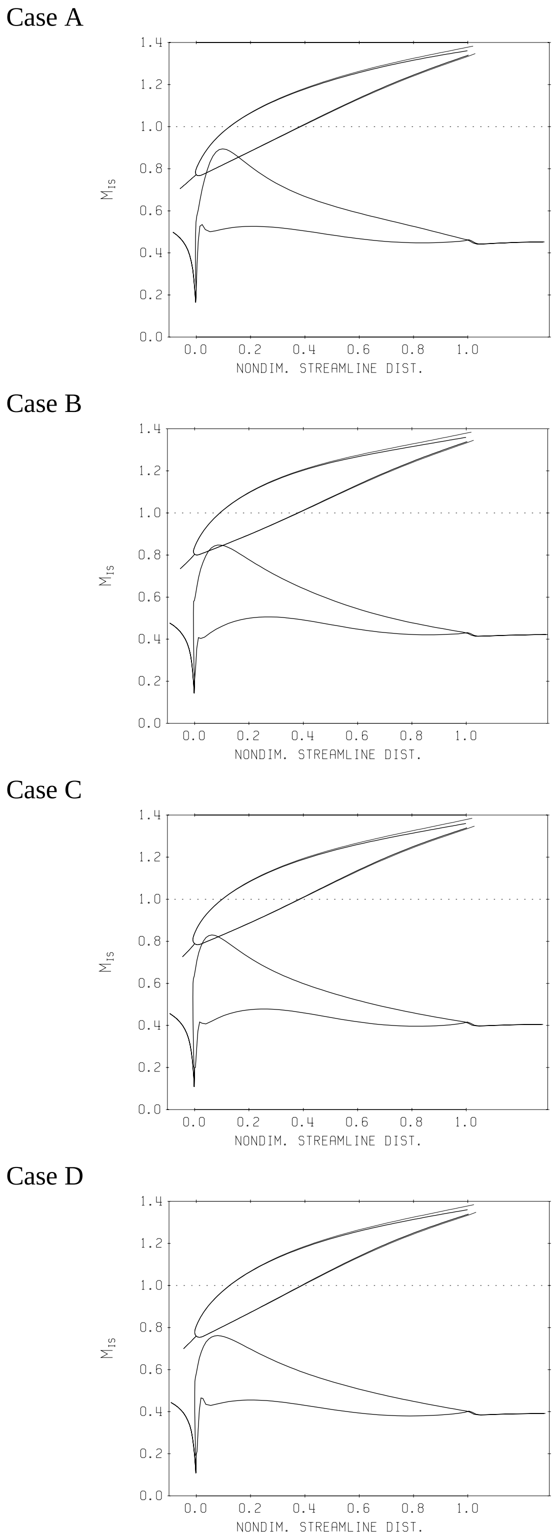
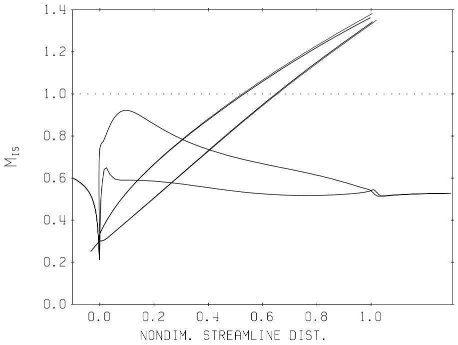
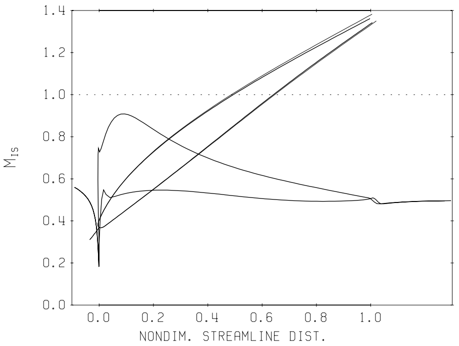


Figure 7. Airfoil shapes and Mach number distribution for hub streamline

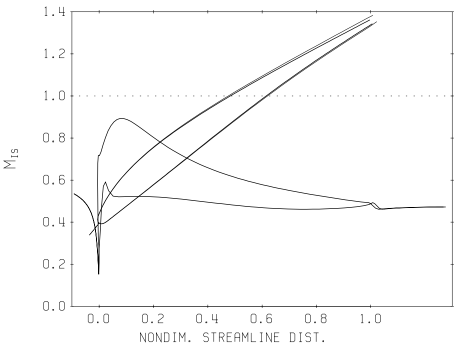
Case A



Case B



Case C



Case D

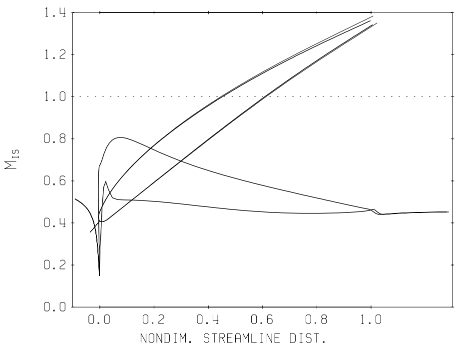
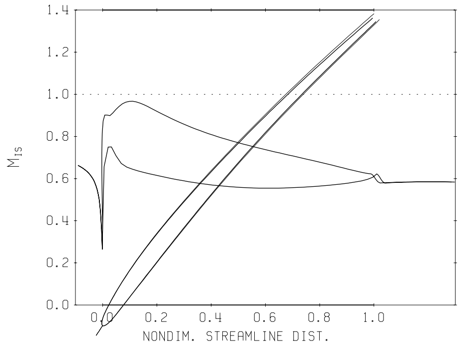
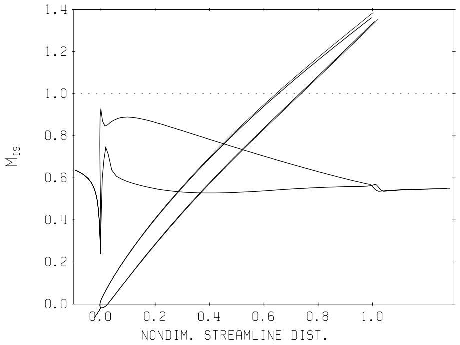


Figure 8. Airfoil shapes and Mach number distribution for mid-span streamline

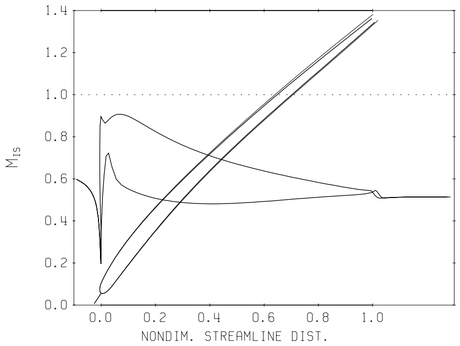
Case A



Case B



Case C



Case D

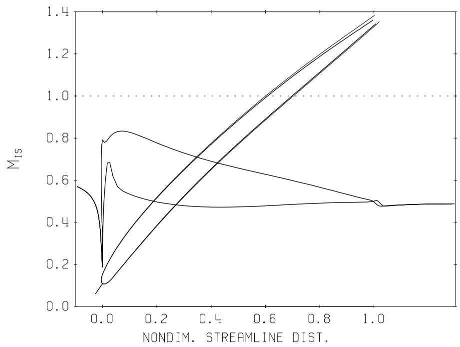


Figure 9. Airfoil shapes and Mach number distribution for tip streamline

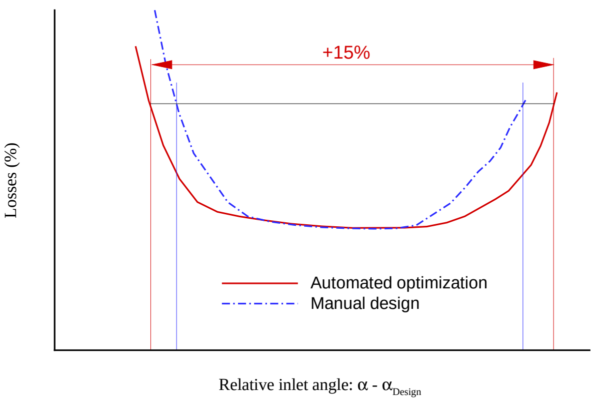


Figure 10. Loss polars for mid-span section, Case A

CONCLUSIONS AND OUTLOOK

This paper presents a automated optimization procedure for sub-sonic compressor blades. Predictions show that the optimized blades have low aerodynamic losses, sustain aerodynamic and centrifugal forces, and avoid critical eigenfrequencies. In addition they show an increased operating range compared to the manual design, while the aerodynamical losses at the design operating point are about equal. The increased operating range is realized by a more pronounced front loading compared to conventional CDA airfoils.

Clearly the quality of the optimization depends strongly on the quality of the prediction models which were used. At this stage, an experimental verification of the new designs or a detailed 3D CFD study has not been undertaken yet. It must be mentioned, however, that the prediction tools are validated standard tools which are used during the manual design process. Furthermore, the effect of front-loaded design on the increased operating range has been experimentally confirmed for profile sections (see Küsters et al. (2000)).

The optimization is performed using Evolution Strategies, which are both fast and robust. An optimization of a blade requires the computation of about 4.000 designs and runs roughly 1/2 day on a four-processor Linux cluster. The robustness is underlined by optimizing four rotor rows of the compressor. Even though optimization runs were started from random initial blades, all obtained designs show similar loading distributions (i.e. profile shapes), thus, the optimization algorithms converges robust to similar optima. In this optimization approach, no gradient information is necessary and the expensive task of formulating an adjoint approach is not necessary.

Ongoing research at the Alstom Power Technology Center aims at a further reduction in computing time by combining response surface techniques with Evolution Strategies. Future investigations will also focus on an assessment of secondary endwall losses for front-loaded blades.

ACKNOWLEDGMENT

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