Exercise 11
Particle Methods and MPI

High Performance Computing for Science and Engineering I

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CSElab
Computational Science & Engineering Laboratory
http://www.cse-lab.ethz.ch

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Point vortex

vorticity

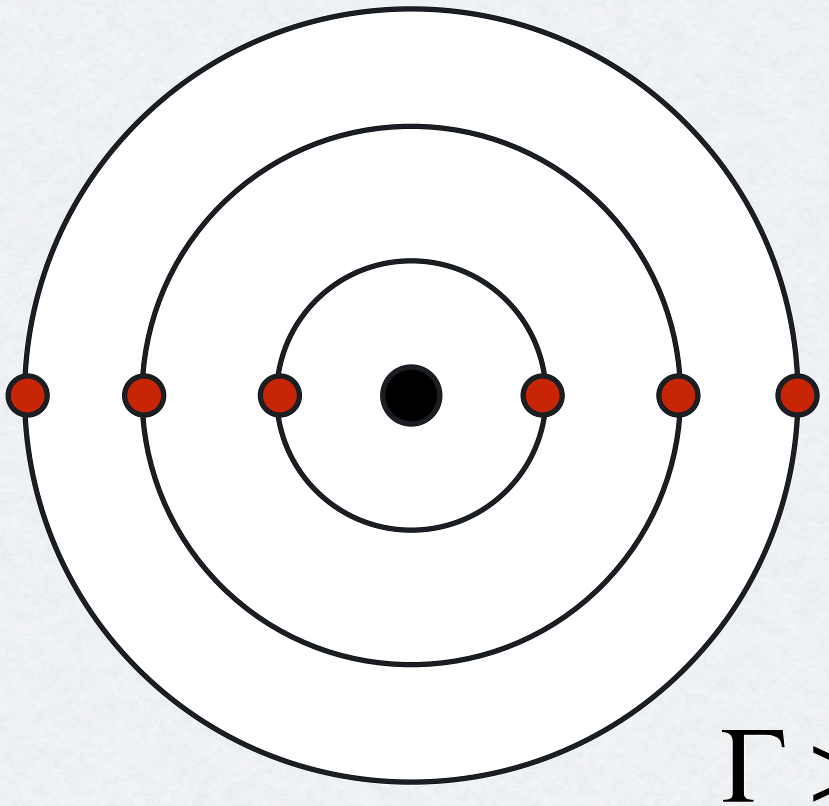
$$\omega(\mathbf{x}) = \Gamma\,\delta(\mathbf{x})$$

velocity $\mathbf{u} = (u, v)$

$$u(\mathbf{x}) = \frac{\Gamma}{2\pi}\frac{-y}{x^2 + y^2}$$

$$v(\mathbf{x}) = \frac{\Gamma}{2\pi}\frac{x}{x^2 + y^2}$$

Γ - circulation



$\Gamma > 0$

Particles as Point Vortices

vorticity

$$\omega(\mathbf{x},t) = \sum_{n=0}^{N-1} \Gamma_n \delta(\mathbf{x} - \mathbf{x}_n)$$

Γ_n - circulation

$\mathbf{x}_n(t)$ - position

velocity

$$\mathbf{u}(\mathbf{x},t) = \sum_{n=0}^{N-1} \mathbf{u}_n(\mathbf{x},t)$$

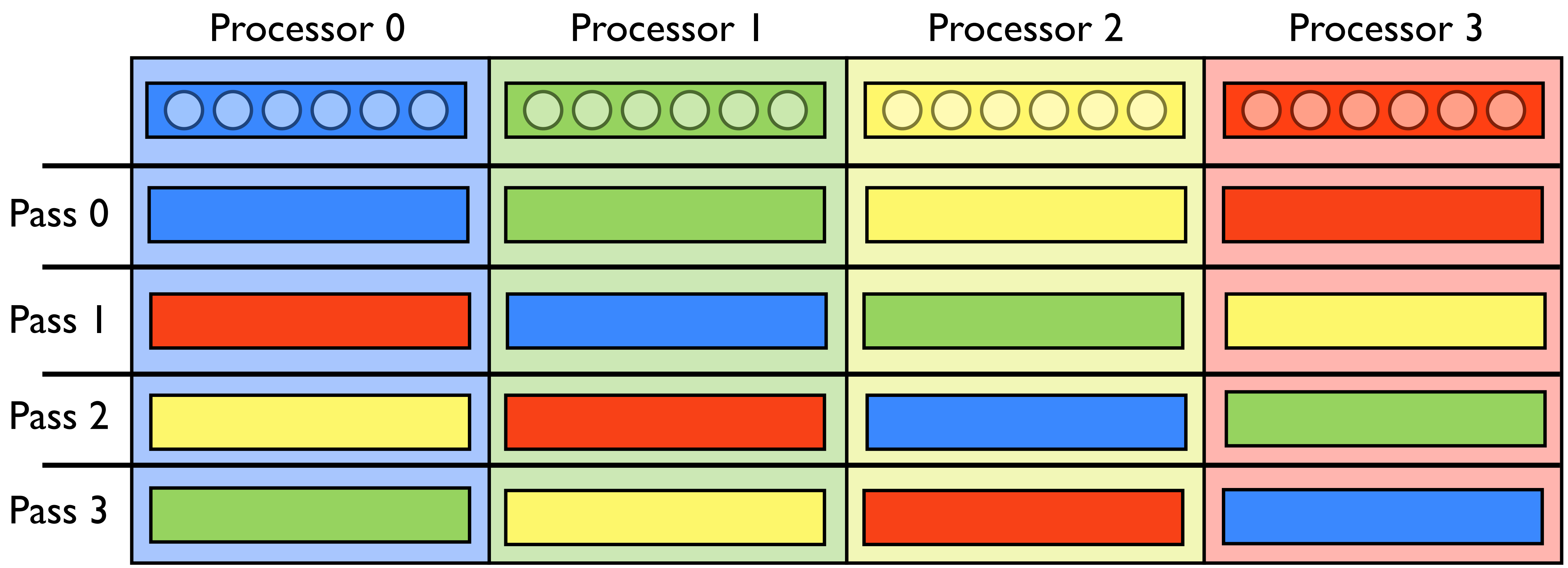
$$u_n(\mathbf{x},t) = \frac{\Gamma_n}{2\pi} \frac{-(y - y_n)}{|\mathbf{x} - \mathbf{x}_n|^2}$$

$$v_n(\mathbf{x},t) = \frac{\Gamma_n}{2\pi} \frac{x - x_n}{|\mathbf{x} - \mathbf{x}_n|^2}$$

motion

$$\frac{d}{dt}\mathbf{x}_n = \sum_{\substack{k=0 \\ k \neq n}}^{N-1} \mathbf{u}_k(\mathbf{x}_n, t)$$

Multi-pass


Processor 0
Processor 1
Processor 2
Processor 3
Pass 0
Pass 1
Pass 2
Pass 3

Paraview

```
x, y, g
0, 0, 0
1, 0, 1
0, 1, 2
1, 1, 3
```

- write csv-file
- open all files at once
- apply filter “Table to Points”
- apply filter “Glyph” to display points as spheres or color them by vorticity