Vorticity flux control for a turbulent channel flow

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A feedback control algorithm using wall only information has been applied in simulations of low Reynolds number (Re=180) turbulent channel flow. The present control scheme is based on the manipulation of the vorticity flux components, which can be obtained as a function of time by measuring the instantaneous pressure at the wall and calculating its gradient. The strength of the unsteady mass transpiration actuators can be derived explicitly by inverting a system of equations whose terms depend on the relative locations of the sensors and actuators. The results of the simulations indicate a large (up to 40%) drag reduction. Moreover it appears that using the present methodology open-loop control laws can be devised. © 1999 American Institute of Physics.

The active control of turbulent flows is gaining recognition as a possible means for greatly improved performance of aerospace and marine vehicles. Along with small and robust sensors and actuators, simple yet effective control algorithms, which are based on measurable flow quantities, are needed to make active feedback control of turbulence a reality. An algorithm for active feedback turbulent flow control (herein referred to as opposition control) was first introduced by Choi, et al.1 In the opposition control approach, the vertical motion of the turbulent flow near the wall is counteracted by an opposing blowing/suction distribution of velocity on the wall. Using this technique a 25% drag reduction was obtained by counteracting the velocity field sensed at y+≈15. Though the opposition control algorithm is simple and effective, it has the drawback that it requires measurements inside the flow domain. In order to alleviate this difficulty, Lee et al.,2 employed a neural network to construct a simple feedback control algorithm using information only at the wall. Their methodology was shown to reduce skin friction by about 20%. Recently, research in the area of active flow control has produced a number of efficient active control strategies based on the exploitation of underlying physical mechanisms (Jimenez,3 Schoppa and Hussain4) or by employing concepts from control theory (Lee et al.,5 Cortelezzi and Speyer6).

In this Letter results are presented from the application of a simple feedback control algorithm based on the measurement and manipulation of the wall vorticity flux. The strength of the unsteady mass/blowing actuators is determined explicitly by inverting a system of linear equations. Application of the control scheme to low Reynolds number turbulent channel flow produced drag reduction of up to 40% using wall information only. Moreover it appears that using the present methodology open-loop control laws can be devised.

We consider a Cartesian coordinate system and flow over a flat wall identified with the xz plane, normal to the y axis. For an incompressible viscous flow over a stationary wall, the vorticity flux is directly proportional to the pressure gradients, as the momentum equations reduce at the wall to

\[ \nu \left( \frac{\partial \omega_y}{\partial y} \right)_w = \frac{1}{\rho} \left( \frac{\partial P}{\partial z} \right)_w, \]

\[ \nu \left( \frac{\partial \omega_z}{\partial z} \right)_w = \frac{1}{\rho} \left( \frac{\partial P}{\partial x} \right)_w, \]

where \( P \) is the pressure and \( \omega_y \) and \( \omega_z \) are the streamwise and spanwise vorticity components. Note that the flux of the wall normal vorticity, \( \omega_x \), may be determined from the kinematic condition \( \nabla \cdot \mathbf{\omega} = 0 \).

Gad-El-Hak7 had shown that the vorticity flux can be affected by wall transpiration as well as by wall-normal variation of the kinematic viscosity (\( \nu \)) as a result of surface heating, film boiling, cavitation, sublimation, chemical reaction, wall injection of higher/lower viscosity fluid, or in the presence of shear thinning/thickening additive.

However an explicit formulation is necessary in order to determine the actuator strengths necessary to induce a desired vorticity flux at the wall. The formulation of the feedback control vorticity flux algorithm is presented in Koumoutsakos.8 The pressure field is sensed at the wall and its gradient (the wall vorticity flux) is calculated. Unsteady blowing/suction at the wall is the actuating mechanism and its strength is calculated explicitly by formulating Lighthill’s8 mechanism of vorticity generation at a no-slip wall.

For the purposes of our control scheme we consider a series of vorticity flux (or equivalently pressure gradient) sensors on the wall at locations \((x_i, z_i), i = 1, 2, 3, \ldots, M\).

We can explicitly determine the actuator strengths necessary to achieve a desired vorticity flux profile at the wall at a time instant, \( k \), by solving the linear set of equations:

\[ B u_k + X_{k-1} = D_k, \]

where \( D_k = (\partial \omega^{k-1}_y / \partial y) (x_1, z_1), (\partial \omega^{k-1}_y / \partial y) (x_2, z_2), \ldots, (\partial \omega^{k-1}_y / \partial y) (x_M, z_M) \) is an \( M \times 1 \) vector of the desired vorticity flux at the sensor locations, \( X_{k-1} = (\partial \omega^{k-1}_x / \partial y) \times (x_1, z_1), (\partial \omega^{k-1}_x / \partial y) (x_2, z_2), \ldots, (\partial \omega^{k-1}_x / \partial y) (x_M, z_M) \) is an \( M \times 1 \) vector of the measured vorticity flux at the sensor locations and \( u_k = (q'_1(x'_1, z'_1), q'_2(x'_2, z'_2), \ldots, q'_M(x'_M, z'_M)) \) is an \( N \times 1 \) vector of source strengths at the actuator locations, \( B \) is an \( M \times N \) matrix whose elements \( B_{ji} \) are a function of the relative locations of the sensors (\( j \) index).
and actuators \((i \text{ index})\), and are calculated by evaluating over each actuator element the integral \(I_{ji}\) and its gradient, where

\[
I_{ji} = \int_{x_i - \Delta x/2}^{x_i + \Delta x/2} \int_{z_j - \Delta z/2}^{z_j + \Delta z/2} q(x_i, z_j) dx \, dz
\]

The unknown source/sink strengths are determined by solving the system in Eq. (2).

We may distinguish between in-phase control (implying enhancement of the wall vorticity flux) by selecting \(D_k = 2X_{k-1}\) and out-of-phase control (implying cancellation of the induced vorticity flux) by selecting \(D_k = 0\).

The present technique gives us the flexibility to adapt the actuator strengths to specific constraints. In the present calculations the requirement of zero net mass flux is easily incorporated in the above scheme by appropriately adjusting matrix \(B\). A square, invertible matrix is always possible by diagonally modifying the number of sensors and actuators. The simplicity of the present scheme allows for a number of different placements of sensors and actuators. Here we chose the locations of sensors and actuators to be collocated. Physically this may be understood as an advantageous situation as the sensors are able to detect the vorticity field induced by the actuators which allows the control scheme to suitably compensate for it.

Simulations of the model problem of vortex dipole-wall interactions\(^8\) have revealed that the present control scheme can drastically alter these interactions. In-phase control results in the “trapping” of the primary vortices by the enhanced secondary vorticity field. The out-of phase control has resulted in the absorption of the impinging dipole and the establishment of small oscillating vortical structures over the wall. These structures are maintained by the present algorithm as the system constantly reacts to the production of the vorticity induced by the actuators.

We present here results from the application of this vorticity-flux control algorithm on a low Reynolds number turbulent channel flow \((Re_z = 180)\).
in the inner layer of the wall. These spanwise vortical rollers result in the formation of positive and negative shear stresses at the wall. The spanwise correlation of the near wall structures persist until about $y^+ = 15$, beyond which the influence of the wall is not discernible in the flow field. Moreover the regularity in the resulting actuator strengths (Fig. 2) suggest that it is possible to devise open-loop control laws using the present methodology. The elimination of streaks and the possibility to devise open-loop control laws using the present methodology.11 The elimination of streaks and the disruption of the near wall processes by the establishment of the particular vortical “rollers” resulted in skin friction drag reduction on the order of 40% (Fig. 3).

It should be noted that in-phase control of the spanwise vorticity flux did not result in a significant mean drag reduction or increase. Again, in this case, it appears that the flow organizes itself in the spanwise direction, but this organization is not stable and is not maintained by the feedback control scheme in this particular sensor-actuator configuration.

The results of these simulations indicate that the formation of spanwise coherent structures in the near wall region can “shield” the wall from the bulk flow, thus significantly altering drag inducing mechanisms such as ejections and sweeps in the near wall region. Further studies are presently ongoing in order to identify the physical mechanisms that lead to this significant drag reduction in the case of out-of-phase control and in the spanwise organization of the near wall structures in both cases.

It should be emphasized that Eq. (2) is used in order to determine the strength of the actuators whereas Eq. (4) is an additional equation that determines how the controlled and measured components are related. When both wall parallel vorticity flux components are being controlled the right-hand side of Eq. (2) becomes a $2M$ by 1 matrix. The 2 by 2 matrix of coefficients in Eq. 4 can be a priori defined by the user or through an optimization process for drag reduction/increase which is a subject of ongoing investigations. In particular, setting $b = c = d = 0$ and $a = \pm 1$ (i.e., adjusting the streamwise flux by in/out of phase with the measured spanwise vorticity flux) results also in significant drag reduction and in flow patterns where the contours of the vorticity flux are inclined with respect to the streamwise direction of the flow.

In order to assess the practical implications of the proposed algorithm we outline some research efforts associated with measurements of the vorticity flux. Experimental measurements of the wall vorticity flux in a turbulent flow have been reported by Andreopoulos and Agui.12 Their experiments demonstrated that the major contributions to the vorticity flux come from the uncorrelated part of the pressure signals, at two adjacent locations, which contain a wide range of vortical scales. As the degree of correlation is smaller between the small scales their contribution to the vorticity flux is more pronounced. This imposes a severe requirement on the spatial resolution of the pressure gradients/vorticity flux measurements. Practical applications would require actuators and sensors with sizes on the order of 50 $\mu m$ and actuator frequencies of 1 MHz. Recent advances in micropressure sensor fabrication technology14 give us an opportunity to overcome these difficulties. Löfdahl et al.15 presented measurements in a two-dimensional flat plate boundary layer with a resolution of eddies with wave numbers less than ten viscous units by using microscopic silicon pressure transducers. It appears that by using this new technology one may be able to describe in detail physical processes in terms of the wall vorticity and the wall vorticity flux. This in turn may lead to a practical implementation of the present control scheme.

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**References**

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