On estimating the aerodynamic admittance of bridge sections by a mesh-free vortex method

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ARTICLE INFO

Article history:
Received 16 March 2015
Received in revised form 11 August 2015
Accepted 12 August 2015

Keywords:
Bridge aerodynamics
Aerodynamic admittance
Buffeting response
Discrete vortex method
Stochastic turbulence generation

ABSTRACT

A stochastic method of generating a synthetic turbulent flow field is combined with a 2D mesh-free vortex method to simulate the effect of an oncoming turbulent flow on a bridge deck cross-section within the atmospheric boundary layer. The mesh-free vortex method is found to be capable of preserving the a priori specified statistics as well as anisotropic characteristics of the synthesised turbulent flow field. From the simulation, the aerodynamic admittance is estimated and the instantaneous effect of a time varying angle of attack is briefly investigated. The obtained aerodynamic admittance of four aerodynamically different bridge sections is compared to available wind tunnel data, showing good agreement between the two.

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1. Introduction

In the design phase of large suspension bridges many resources are normally used to conduct extensive wind tunnel tests to prevent structural failure caused by aerodynamic forces. Various experimental methods are used to determine the influence of static, periodic and stochastic aerodynamic forces on the bridge, and how these excite certain structural responses of the bridge. Due to a required high structural stiffness, bridge decks are typically box-shaped and hence aerodynamically bluff bodies. The result is a highly complex flow around the bridge deck, which can cause a number of aero-elastic phenomena to occur under different conditions. Aerodynamically bluff bodies are generally more sensitive to flow separation, and thus a significant change in the aerodynamic forces may be observed when varying the angle of attack. Hence the effect of turbulence in the oncoming flow becomes important when evaluating the complete aerodynamic performance of the bridge section, as the turbulence results in a fluctuating effective angle of attack.

The effect of a turbulent oncoming flow on the aero-elastic interactions of the bridge is indeed non-trivial. On one hand the turbulent fluctuations of the flow will introduce a stochastic aerodynamic force which will disturb any periodic excitation and thus stabilise the bridge against flutter. On the other hand the stochastic fluctuations may themselves be a cause to unpredicted high aerodynamic forcing on the bridge. As turbulence consists of eddies of multiple scales that are transported by the flow, the sampled flow velocity at a fixed point will show a time history which contains a large band of frequencies at different energy levels. Therefore, with a strong analogy to ocean waves, situations occur where the turbulent eddies at different flow scales become instantaneously synchronised resulting in a fluctuation that is even larger than the amplitude of the largest eddies in the flow. When subject to such super-scale fluctuations the bridge section will experience sudden large change in the aerodynamic forcing and a vortex formation on the leading edge of the bridge may occur. Vortex formation on the leading edge is of significant concern as it produces strong gusts on the traffic lane which may be dangerous for large vehicles travelling on the bridge deck.

Prendergast and McRobie (2006) and Prendergast (2007) presented a vortex method to simulate an oncoming turbulent flow in bluff body aerodynamics. The flow was simulated using the Discrete Vortex Method (DVM) implementation VXFlow by Morgensthal (2002). The oncoming turbulent flow was implemented by synthesising a time varying turbulent velocity field by a stochastic method, originally proposed by Shinozuka and Jan (1972), on the corner-points of a single column of mesh cells upstream of the bridge section. The circulation of each cell of the velocity field was then calculated and included in the DVM simulation by seeding the circulation as vortex particles throughout the simulation.
Rasmussen et al. (2010) used a similar approach, based on the DVM implementation DVMFLOW by Walther (1994) and Walther and Larsen (1997), to obtain an extended aerodynamic analysis of the effect of a turbulent oncoming flow. Emphasis was placed on the spectral transfer functions between the turbulent velocity fluctuations of the oncoming wind and the resulting buffeting forces acting on the bridge section. This transfer function is referred to as the aerodynamic admittance function. Rasmussen et al. (2010) showed that the aforementioned method is able to successfully calculate the aerodynamic admittance function of a flat plate.

The concept of aerodynamic admittance employed in the present paper represents the chord wise filtering action of the solid deck section on the incoming turbulence. This definition is concordant with the two-dimensional (2D) classical assumption that the turbulent wind impacting onto a given span wise section creates aerodynamic forces proportional to the steady state load coefficients at this section only and as a consequence the span wise coherence of the aerodynamic forces is identical to the span wise coherence of the oncoming turbulence. Recent research has demonstrated that this is not the case for common bridge decks for which the span wise forces are found to be much more correlated than the oncoming turbulence although of less magnitude than expected from the classical assumption (Larose, 2003). Strip theory which splits the aerodynamic action of the turbulence in a chord wise (aerodynamic admittance) and a span wise component (root coherence of turbulence) is often employed in common commercial gust loading (buffeting) calculations for want of better models thus 2D aerodynamic admittance functions simulated in the present paper remain interesting to bridge designers.

In this paper, we extend the validation of the flat section method of Rasmussen et al. (2010) towards a practical application in bridge aerodynamics. The aerodynamic admittance function of four different bridge sections are investigated and compared with available experimental data. The bridge sections which are shown in Fig. 1 represents a selection of the most common bridge deck types used in bridge design: the mono box bridge (Great Belt East bridge), the double deck truss bridge (Øresund bridge), the plate girder bridge (Busan–Geoje bridge) and the twin box bridge (Stonecutters Bridge).

In this work only a brief outline of the method is presented and the reader is referred to Rasmussen et al. (2010) and Rasmussen (2011) for a detailed description and validation of the applied numerical method.

2. Numeric method

2.1. The discrete vortex method

The flow is simulated using the two-dimensional DVM implementation DVMFLOW by Walther (1994) and Walther and Larsen (1997). Here the velocity–vorticity formulation of the Navier–Stokes equation is solved in a Lagrangian frame of reference by simulating computational particles which represents an elementary distribution of vorticity referred to as a vortex blob. Hence, as vorticity is a material property which is advected with the flow, the particle position \( \mathbf{x}_p \) is solved in the Eulerian frame of reference by

\[
\frac{d}{dt} \mathbf{x}_p = \mathbf{v}(\mathbf{x}_p).
\]

The velocity field \( \mathbf{v} = (u, v, 0) \) is obtained from the vorticity field \( \omega \equiv \nabla \times \mathbf{v} \) by solving the inverted kinematic relation for an incompressible flow, where \( \nabla \cdot \mathbf{v} = 0 \), by which

\[
\mathbf{v} = - \nabla \omega.
\]

The vorticity only has the out-of-plane component in a 2D flow i.e. \( \omega = (0,0,\omega_z) \). Eq. (2) is recognised as a Poisson equation and can thus be solved for an arbitrary point \( \mathbf{x}_p \) using a Green’s function solution

\[
\mathbf{v}(\mathbf{x}_p) = \int_{\mathbb{R}^2} \mathbf{K}(\mathbf{x}_p - \mathbf{x}) \omega(\mathbf{x}) \, d\mathbf{x}
\]

where \( \mathbf{K} \) is the 2D Green’s function for Eq. (2) which for a second order Gaussian regularised vortex blob is given as

\[
\mathbf{K} = - \frac{1}{2\pi r} \left( 1 - \exp\left( - \frac{r^2}{2\sigma^2} \right) \right) \left( \frac{y_p - y}{y_p - x_p} \right) \frac{x_p - x}{y_p - x_p}.
\]

Here \( r = |\mathbf{x}_p - \mathbf{x}| \) and \( \sigma \) is the blob radius which in the present implementation is related to the discretisation of the geometry by \( \sigma / \delta l = 2 \) where \( \delta l \) is the length of the discretisation segment. For a particle discretisation Eqs. (3) and (4) can be combined as

\[
\mathbf{v}(\mathbf{x}_p) = \mathbf{V} - \sum_{i=1}^{N_p} \int_{\mathbb{R}^2} \left( 1 - \exp\left( - \frac{r_i^2}{2\sigma^2} \right) \right) \left( \frac{y_p - y_i}{y_p - x_p} \right) \frac{x_p - x_i}{y_p - x_p}.
\]

\( N_p \) is the number of particles in the flow, \( \mathbf{V} = (U, 0) \) the free-stream velocity, and \( \int I \) is the circulation of the particles which super-positioned represents the integral of the vorticity field. To reduce the computational load, Eq. (5) is evaluated using the fast multi-pole method (Carrier et al., 1988).

The incompressible 2D Navier–Stokes equation is solved in the following velocity–vorticity form:

\[
\frac{D}{Dt} \omega = \nu \nabla^2 \omega,
\]

where \( \nu \) denotes the kinematic viscosity of the fluid. It is seen that Eq (6) is a diffusion equation which may be solved by a stochastic random walk method (Chorin, 1973). Hence, the diffusion of vorticity (Eq. (6)) may be included in the trajectory equation (Eq. (1)) by introducing a random walk in which the trajectory equation

Fig. 1. Cross-sections of the four bridge decks (scaled to unity chord) which are investigated in the present study. (a) Great Belt East bridge, mono box girder bridge, (b) Øresund bridge, double deck truss bridge, (c) Busan–Geoje bridge, plate girder bridge, and (d) Stonecutters bridge, twin box girder bridge.
becomes
\[ \frac{d}{dt} \mathbf{x}_p = \mathbf{v}_p + \sqrt{2\nu} \mathbf{q}, \]
(7)

Here \( \mathbf{q} \) is a Gaussian distributed random number with a zero mean and a unit standard deviation, and \( \Delta t \) is the time step size used in the simulation.

2.2. Simulating the effect of turbulent upstream flow

Following the work of Prendergast and McBride (2006), Prendergast (2007) and Rasmussen et al. (2010) the effect of turbulent oncoming flow is simulated by releasing vortex particles upstream of the bridge section to induce velocity fluctuations in the oncoming flow. This is done by seeding pre-generated vortex particles in a single column located at a specified distance upstream of the bridge section. A new column of particles is released at a specified time frequency during the simulation. Hence the simulated velocity fluctuations may contain frequencies up to half the seeding frequency (Nyqvist sampling theorem). As the simulation progresses the seeded vortex particles create a particle cloud consisting of multiple interacting vortex particles which emulates turbulent eddies of multiple scales. As the bridge becomes immersed in the particle cloud it will interact with the seeded particles and thus the effect of a turbulent oncoming flow is included in the simulation and the resulting buffeting forces may be extracted from the data.

The pre-generated \( N_c \) vortex particles, which are seeded at a specified time interval \( \Delta t_{\text{gen}} \) during the simulation, are constructed by defining a release ladder consisting of a \( N_c = 2 \times \left( N_x + 1 \right) \) array of cell corner points with a uniform spacing of \( \Delta x = \Delta t_{\text{gen}} \) with the particles located at the cell centre as seen in Fig. 2. In the corner points of the release ladder, two-component time signals of a turbulent velocity field are generated by a stochastic method originally proposed by Shinozuka and Jan (1972) in order to reproduce the correlation characteristics of a turbulent flow.

Generating a multiple series of correlated random numbers by a stochastic simulation is done by first defining a correlation matrix which gives the correlation between each of the random number series i.e. each of the corner points of the release ladder. A random correlated field \( \gamma(\mathbf{x}, t) \) which is a function of both space and time may be discretised as a multivariate process dependent on time only, hence
\[ \gamma(\mathbf{x}, t) \rightarrow \{ \gamma_1(t), \gamma_2(t), \ldots, \gamma_{N_c}(t) \} \]
(8)

where the sub-index refers to the corner-point positions sketched in Fig. 2. The correlated random numbers are generated by convolving a white noise variable with a function which generates the desired correlation. For a multivariate process this may be written as
\[ \gamma(t) = \int_{-\infty}^{\infty} H_{ij}(t - \tau) \phi_i(\tau) d\tau, \quad (i, j) = \{1, 2, \ldots, N_c\}. \]
(9)

Here \( \phi_i(t) \) is a random white noise time signal at point \( \mathbf{x}_i \) and \( H_{ij}(t - \tau) \) is a matrix which generates the correlation between point \( \mathbf{x}_i \) and \( \mathbf{x}_j \) given a time lag of \( \tau \). The convolution of Eq. (9) is effectively done spectrally by fast Fourier transforms where the spectral functions of \( H_{ij}(t) \) which is denoted by \( \tilde{H}_{ij}(k) \) can be determined from the spectral correlation matrix \( \tilde{C}_{ij}(k) \). The spectral correlation matrix is defined by the given spectral power density function \( S(k) \), which gives the statistical characteristics of the turbulent kinetic energy at frequency \( k = 2\pi/T \), combined with a coherence function (Davenport, 1968; Solari, 1987; Rossi et al., 2004) which correlates the points to give
\[ \tilde{C}_{ij}(k) = S(k) \exp \left( -\frac{k}{2\pi} \left( \frac{c_x^2}{U} (X_i - X_j)^2 + \frac{c_y^2}{U} (Y_i - Y_j)^2 + 2\pi i (X_i - X_j) \right) \right) \]
(10)

where \( c_x \) and \( c_y \) are the directional decay parameters of turbulent eddies and \( i \) is the imaginary unit. The effect of free-stream advection is included by introducing a phase shift which is determined using Taylor’s frozen turbulence hypothesis as the imaginary part of Eq. (10). The relation between \( \tilde{C}_{ij}(k) \) and \( \tilde{H}_{ij}(k) \) is given by the spectral correlation function by inserting Eq. (9)
\[ \tilde{C}_{ij}(k) = \tilde{H}_{ij}^* \quad \tilde{H}_{ij} = \left( \tilde{H}_{ik} \tilde{H}_{kj}^* \right) = \tilde{H}_{ik} \tilde{H}_{kj}^T \]
(11)

where the superscripts * and T denote the complex conjugate and the transpose, respectively. Here the correlation properties of the white noise variable are used which in spectral form is
\[ \tilde{\phi}_k \tilde{\phi}_l^* = \begin{cases} 1 & \text{for } k = l \\ 0 & \text{for } k \neq l \end{cases} \]
(12)

From Eq. (10) we see that \( \tilde{C}_{ij} \) is a Hermitian matrix (i.e. \( \tilde{C}_{ij} = \tilde{C}_{ij}^T \)) which enables the determination of \( \tilde{H}_{ik} \) to be performed by the Cholesky decomposition of \( \tilde{C}_{ij} \). The matrix \( \tilde{C}_{ij} \) is positive definite in the present application which ensures a unique decomposition. Once \( \tilde{H}_{ik} \) is defined, it is straightforward to calculate a series of correlated random variables \( \gamma(t) \) by Eq. (9).

The process is performed independently for both components of the velocity field i.e. \( \gamma = \{u, v\} \) without any cross-correlation between the components. For the spectral energy density functions \( S(k) \) we use the semi-empirical spectral functions \( S_u(k) \) and \( S_v(k) \) which were proposed by the ESDU (1993, 2001) for measured

![Fig. 2. Conceptual sketch of the release ladder (here \( N_x = 8 \) and \( N_c = 3 \) upstream of the bridge section. The velocity signal is generated at the \( N_c \) corner points of the release ladder (grey/red crosses) and the \( N_x \) vortex particles at the cell centre (grey/blue dots).](image)
turbulence in the atmospheric boundary layer and are summarised in Appendix A. The ESDU spectra may be constructed by defining the turbulence intensity, the atmospheric boundary layer thickness, the height above ground, and the surface roughness length. For the present implementation these input parameters are the same for all points of the release ladder.

The approach outlined above implies that the resulting velocity field does not represent a solution to the Navier–Stokes equations hereunder the incompressibility condition of \( \nabla \cdot \mathbf{v} = 0 \). The generated field is a synthetic flow field in which the fluctuations of the two velocity components possess the same statistical characteristics as the corresponding two velocity components of a 3D turbulent flow field measured in the atmospheric boundary layer. However the extensive analysis presented in Prendergast and McRobie (2006), Prendergast (2007), Rasmussen et al. (2010), and Rasmussen (2011) showed that when the generated velocity field is converted to circulation and simulated in the discrete vortex method it is indeed possible to admissibly reproduce the statistical characteristics of the input energy density spectra.

The generated velocity field is converted to circulation of a cell centred particle for each time step by integrating the velocity of the surrounding cell corner points by

\[
\Gamma = \oint_{c} \mathbf{v}(x) \cdot ds
\]

(13)

Using the trapezoidal rule, the velocities of the corner points are integrated by assuming a linear variation between the corner points of the release ladder. To correct for the varying distance \( r \) from the cell edge to the centre of the cell, the circulation is corrected in magnitude by a factor of \( x/2 \) as proposed by Prendergast (2007).

2.3. The boundary element method

The bridge section is simulated by using a boundary element method (Wu, 1976; Walther, 1994). Here the geometry of the bridge section is discretised by a finite number of line segments referred to as panels and the boundary conditions of the fluid–solid interface is enforced by introducing linearly varying vortex sheets at each of the panels. The strength of the vortex sheets is initially unknown and provides a sufficient number of degrees of freedom in order to obtain a flow solution where the velocity component perpendicular to the surface of the geometry is zero i.e. the no-penetrating-flow boundary condition.

The solution is formally obtained by solving a linear system of equations with the added constraint of a zero sum circulation of the entire domain (Walther and Larsen, 1997) i.e. Kelvin’s condition of constant total circulation. However, the circulation of the upstream particles that are seeded in order to simulate an oncoming turbulent flow as described in Section 2.2 introduces a non-zero net circulation to the domain. This is accounted for by adjusting the constraint of total circulation to add up to the accumulated circulation \( \Gamma(t) \) which is introduced by the seeded turbulence particles i.e.

\[
\sum_{i=1}^{N} \Gamma_i = \Gamma(t)
\]

(14)

as was proposed by Rasmussen et al. (2010). The vortex sheets are converted into a number of vortex particles at each panel which are diffused into the flow. For this, the random walk diffusion of the particles created from the vortex sheet is adjusted to give a one-sided diffusion in the direction normal to the panel. Particles located near the solid boundary may be moved into the solid due to the discrete time stepping or the random walk diffusion model. Such particles are removed from the simulation after which their circulation is implicitly included when solving the boundary conditions in the next time step due to the constraint of Eq. (14).

Knowing the strength of the vortex sheet \( \gamma \) located at each panel it is possible to calculate the pressure distribution by a simple discretisation of the relation

\[
\frac{1}{\rho} \frac{\partial p}{\partial t} = -\frac{\partial \mathbf{v}}{\partial t} \cdot \mathbf{n}
\]

by which \( \delta p_i = -\rho \frac{\partial \gamma}{\partial x} \)

(15)

Here \( \rho \) is the fluid density and \( \delta l_i \) denotes the length of the \( i \)th panel. Given the pressure distribution the forces and moments are simply calculated by summing the contribution of each panel. The total forces and moments including viscous shear are computed from the vorticity moments cf. Wu (1978) and Walther and Larsen (1997).

2.4. Spectral analysis and the aerodynamic admittance function

Throughout the simulation, time signals of the velocity components \( u \) and \( v \) are sampled at a fixed sample point upstream of the bridge section together with the aerodynamic forces of lift \( L \), drag \( D \) and pitching moment \( M \) acting on the bridge section. From the fluctuating part of the time series the power spectra \( S(k) \) are calculated using Welch’s method (Welch, 1967) with a 50% sample overlap.

The aerodynamic admittance function is a spectral relation between the energy of the fluctuating forces for lift or pitching moment and the energy of the fluctuations of the oncoming velocity field. The calculated aerodynamic admittance function for the lift force \( A_L \) and pitching moment \( A_M \) may be expressed as

\[
A_L(k) = \frac{S_L(k)}{\left( \frac{1}{2} \rho U^2 \right)^2 \left[ 4 \pi^2 \delta L \left( \frac{d \gamma}{d u} \right) + \left( \frac{d \gamma}{d C_{L}} \right)^2 \right] S_V(k)}
\]

(16)

and

\[
A_M(k) = \frac{S_M(k)}{\left( \frac{1}{2} \rho U^2 \right)^2 \left[ 4 \pi^2 \delta M \left( \frac{d \gamma}{d C_{M}} \right) + \left( \frac{d \gamma}{d C_{M}} \right)^2 \right] S_V(k)}
\]

(17)

as proposed by Gu and Qin (2004). Here \( C_L \) and \( C_M \) are the force coefficients per unit length for lift and pitching moment respectively which is defined by

\[
C_L = \frac{L}{\frac{1}{2} \rho U^2 C} \quad \text{and} \quad C_M = \frac{M}{\frac{1}{2} \rho U^2 C^2}
\]

(18)

where the angle of attack \( (\alpha) \) may be estimated by the small angle approximation \( \alpha \approx \tan \alpha = v/U \) when deriving Eqs. (16) and (17).

It is well stated in the literature (Jancauskas, 1986; Hatanaka and Tanaka, 2002; Costa, 2007) that the admittance function of bridge decks deviates from that of thin aerofoil theory, which is also the case with the numerical and experimental results presented in this work. Thin aerofoil theory however provides a good reference for the aerodynamic admittance functions of bluff bodies, and is presented by a simplification of the Sears function proposed by Liepmann (1952)

\[
A_L(k) = A_L(1) = \frac{1}{1 + \pi k U}
\]

(19)

where \( A_L \) and \( A_M \) denotes the aerodynamic admittance functions for lift and pitching moment respectively of a thin aerofoil.
3. Results

3.1. Simulation parameters and numerical set up

The simulations of the four bridge sections were performed using input parameters which represent realistic conditions for a bridge deck within the atmospheric boundary layer. For generating the upstream turbulence particles (Section 2.2) the input velocity power spectra of Eqs. (A.1) and (A.2) was constructed using a frequency discretisation of 4096 frequencies, giving a simulated highest and lowest frequency \( k \) of 5.83 and \( 1.42 \times 10^{-3} \) rad/s, respectively. The physical parameters used in Eqs. (A.1) and (A.2) are chosen to simulate an atmospheric turbulent wind over open landscape, with a turbulence intensity \( I = \sigma_u/U \) of 5% where \( \sigma_u \) is the standard deviation of the vertical velocity component. The surface roughness used to calculate the shear velocity \( u_r \) is set to \( 3 \times 10^{-3} \) m, the thickness of the atmospheric boundary layer \( h = 660 \) m, and height above ground \( y_0 = 70 \) m. To dimensionalise the input and the output of the vortex simulation a chord length of \( c = 30 \) m and a horizontal free stream velocity of \( U = 35 \) m/s are used for all bridge decks. The horizontal free stream velocity is set to increase the distance between the released particles to achieve a wide particle cloud, without adding unnecessary computational load by simply adjusting the number of particles (Rasmussen et al., 2010). It is important that the particle cloud is wide to ensure that the bridge section is fully immersed in the turbulent flow. A convergence of the results was found with 120 particles inserted with a non-dimensionalised distance of \( x_c/\delta = 0.117 \). This produces a wide particle cloud without compromising the density of particles in the flow.

The vortex simulations are performed at a Reynolds number \( \text{Re} = Uc/\nu = 10^4 \). The simulations are performed with 40,000 time steps of \( \Delta t = 0.025 \) corresponding to a dimensional time of 857 s. Before the spectral analysis, the initial 2000 sampled time steps are discarded ensuring that data is sampled only when the bridge is fully immersed in the turbulent flow.

As mentioned in Section 2.3 the boundary element method uses a finite number of linear panels to define each bridge section, on which the surface circulation is created. The Great Belt East bridge was discretised with 200 panels, Øresund bridge with 420 panels, Busan–Geoje bridge with 200 panels, and the Stonecutters bridge with 300 panels.

3.2. Estimation of the static aerodynamic coefficients of the bridge sections

The aerodynamic admittance functions Eqs. (16) and (17) depend on the sampled velocity spectra \( S_u \) and \( S_v \) which are normalised by the static coefficients and their derivatives, assuming a linear variation with \( \alpha \). The dependence of the static coefficients on the angle of attack \( \alpha \) is initially estimated in separate simulations using a laminar oncoming flow at different values of \( \alpha \).

It is seen in Fig. 3 that the static coefficients for most of the bridge sections do not display any form of symmetry around \( \alpha = 0 \). This is because the geometry of the bridge sections are only single symmetric and not double symmetric. The non-linear behaviour of the coefficients indicates that the linear approximation which Eqs. (16) and (17) are based on is valid only in a limited angular range. In Table 1 and Fig. 3 the linear fit of the static coefficients and their derivatives are summarised and visualised respectively.

![Fig. 3. The calculated static coefficients (full) for the lift force (red/black) and the pitching moment (blue/grey) and the corresponding approximated linear fit (dashed).](image)

(a) Great Belt East bridge, (b) Øresund bridge, (c) Busan–Geoje bridge, and (d) Stonecutters bridge.
3.3. The simulated oncoming turbulent flow

The statistical properties of the velocity field induced by the upstream particle cloud is influenced by the presence of the bridge section as well as the boundary of the particle cloud (Rasmussen et al., 2010). In order to diminish this influence and obtain converged velocity power spectra, the velocity was sampled during the simulations at the height of the bridge section, 6 chord lengths upstream of the bridge section and 19 chord lengths downstream of the particle release ladder. The resulting power spectra $S_u$ and $S_v$ of the simulated particle cloud are shown in Fig. 4a and b compared to the input spectra $S_u$ and $S_v$ of Eqs. (A.1) and (A.2) used to generate the turbulence particles by the stochastic method presented in Section 2.2. In Fig. 4c and d the autocorrelation of the two velocity components are shown compared to the ESDU (1993, 2001) reference. The simulated velocity field is statistical consistent for all simulations and is seen to produce a fair agreement with the input spectra though a noticeable deviation is observed for the vertical component at low frequencies. The same deviation is also observed for large time-lags $\tau$ in the autocorrelation function where the resulting integral length scales $L_u = 7.22$ and $L_v = 1.65$ were found compared to $L_u = 7.66$ and $L_v = 0.84$ of the input spectrum cf. Eq. (A.5). Altogether the deviation is noticed to be towards a more isotropic turbulent field than that of the anisotropic input spectra.

This deviation is partly believed to arise when converting the velocity field of the release ladder into an array of vortex particles. As the velocity components are generated independently on the release ladder allowing a divergence in the resulting velocity field, a part of the generated velocity field is discarded as the calculation of the circulation by Eq. (13) disregards any divergence of the velocity field. As a result the total kinetic energy of the vertical fluctuations is observed in Fig. 4b to be increased compared to that of the input spectra. Additionally, the generated velocity field represents only two of the three velocity components of a 3D turbulent flow. Hence by simulating the flow in 2D only, the flow dynamics is constrained in a way that leads to a different energy transfer than that of the 3D flow and thus changes the energy spectra of the turbulent flow. A more thorough investigation of the evolution of the particle cloud is presented by Rasmussen et al. (2010).

Table 1
The static aerodynamic coefficients of lift $C_L$, drag $C_D$ and pitching moment $C_M$ and their approximated derivatives with respect to the angle of attack $\alpha$ at $\alpha = 0$. $k_{sh}$ denote the estimated vortex shedding frequency in rad/s and $St = k_{sh}/U$ the corresponding Strouhal number.

<table>
<thead>
<tr>
<th>Bridge</th>
<th>$C_L$</th>
<th>$C_D$</th>
<th>$C_M$</th>
<th>$\partial C_L/\partial \alpha$</th>
<th>$\partial C_M/\partial \alpha$</th>
<th>$k_{sh}$</th>
<th>$St$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Great Belt East bridge</td>
<td>0.06</td>
<td>0.07</td>
<td>0.03</td>
<td>4.73</td>
<td>1.04</td>
<td>1.43</td>
<td>0.20</td>
</tr>
<tr>
<td>Øresund bridge</td>
<td>0.37</td>
<td>0.21</td>
<td>-0.08</td>
<td>4.23</td>
<td>-0.20</td>
<td>1.80</td>
<td>0.25</td>
</tr>
<tr>
<td>Busan–Geoje bridge</td>
<td>0.13</td>
<td>0.10</td>
<td>-0.01</td>
<td>8.12</td>
<td>0.13</td>
<td>1.11</td>
<td>0.15</td>
</tr>
<tr>
<td>Stonecutters bridge</td>
<td>0.05</td>
<td>0.04</td>
<td>-0.02</td>
<td>2.08</td>
<td>-0.48</td>
<td>2.18</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Fig. 4. The sampled velocity field (red/grey) compared to the ESDU model (ESDU, 1993) used to create the velocity field (black). (a) Velocity power spectrum of the $u$-component, (b) velocity power spectrum of the $v$-component, (c) auto-correlation function of the $u$-component, and (d) auto-correlation function of the $v$-component.
the aerodynamic admittance function could thus be obtained using any velocity field containing fluctuations at a given frequency range of interest.

3.4. Analysis of the simulated flow around the bridge sections

The simulated flow fields around the bridge sections are shown in Fig. 5. Here the instantaneous position and velocity of each vortex particles are plotted. The bridge sections are immersed in the turbulent cloud of particles which is formed by seeding particles upstream of the bridge section as described in Section 2.2. The upstream particle cloud interact with the particles created at the panels of the bridge section resulting in a more irregular vortex shedding than is observed without the upstream seeding of particles.

In a Lagrangian simulation as the DVM the computational elements i.e. the vortex particles are following the trajectory given by the velocity field. Thus it is easy from a flow plot such as those shown in Fig. 5 to identify important flow structures such as recirculation zones and vortex shedding. It is seen in Fig. 5 that all bridge sections have significant vortex shedding from the trailing edge of the sections. The vortex shedding is highly periodic and may cause large bridge deck oscillations if it is undisturbed. This is the case with the Great Belt East bridge where no other flow structures occur but the trailing edge vortex shedding (see Fig. 5a). Consequently the Great Belt East bridge is sensitive to vortex induced vibration as verified in previous wind tunnel tests (Larsen, 1993) and numerical simulations (Larsen and Walther, 1997). For the completed bridge, guide vanes were attached beneath the bridge deck in order to reduce the vortex shedding and ultimately the bridge deck oscillations.

The bridge deck cross section of the Øresund bridge consists of multiple sub-structures as the bridge deck itself is a double deck truss bridge. Each of the sub-structures have an individual vortex shedding (Fig. 5b) which makes the combined vortex shedding of the bridge deck highly irregular. This disturbs the periodic effect of the force associated with vortex shedding making the bridge deck more aerodynamically stable.

The flow image of the Busan–Geoje bridge deck (Fig. 5c) reveals that the girder structure of the bridge deck causes a significant re-circulation zone behind the leading edge girder. At times this re-circulation zone becomes unstable resulting in a large vortex which travels to the trailing edge girder of the bridge section. This kind of flow behaviour evidently creates a time-lagged correlation of the local aerodynamic forcing at the leading and trailing edges of the bridge and is known to cause a pitching motion of the bridge deck (Lawson, 1980).

The Stonecutters bridge deck consists of two relatively thin box-sections. It is seen in Fig. 5d that the large gap between the boxes allows vortex shedding to form on the leading box which highly influences the flow around the trailing box. The wake of the leading box creates an oscillating inflow angle of attack on the trailing box which increase the periodic aerodynamic forcing and may result in leading edge flow separation on the trailing box.

3.5. Estimation of the aerodynamic admittance function

The aerodynamic admittance functions which are calculated from the data extracted from the simulations are shown in Fig. 6 for the lift force and Fig. 7 for the pitching moment along with available experimental data (Strømmen et al., 1996; COWI, 2008) and the aerodynamic admittance functions of thin aerofoil theory (Eq. (19)). The calculated aerodynamic admittance functions are generally different from that of the thin aerofoil theory which is to be expected and show an overall good agreement to the aerodynamic admittance functions of experimental investigations (Strømmen et al., 1996; COWI, 2008). The observed deviations of the aerodynamic admittance function for the lift force at the low frequencies are believed to be caused by different integral lengths in the turbulence fields of the simulation and experiment. As proposed by Larose and Mann (1998) the relation between the vertical integral length scale of the turbulence and the chord length of the bridge section (i.e. \( L_v/c \)) significantly effects the coherence function and thus also the aerodynamic admittance of the lift force. Increasing the relation was showed in Larose and Mann (1998) to increase the aerodynamic admittance function. This corresponds well with the observed deviations in Fig. 6 where the simulated relation was \( L_v/c = 1.65 \) which is believed to be higher than that of the presented wind tunnel experiments. The exact value of \( L_v/c \) obtained in the wind tunnel experiments (Strømmen et al., 1996; COWI, 2008) are however not reported.
For all bridge sections which are investigated in this work a peak is identified in the estimated aerodynamic admittance functions. These peaks are related to the primary vortex shedding of the bridge sections. The vortex shedding occurs at a specific frequency related to the geometry of the bridge section and the dynamic characteristics of the oncoming flow. The vortex shedding frequency of the bridge sections for the current simulation parameters along with the corresponding non-dimensional Strouhal numbers is summarised in Table 1.

Aside from the peak associated with vortex shedding it is seen that the Great Belt East bridge has an aerodynamic admittance which is close to that of thin aerofoil theory for the lift force as well as the pitching moment. The same is found regarding the Busan–Geoje bridge when it comes to the aerodynamic admittance of the lift force but a different behaviour is found for the pitching moment. Here the aerodynamic admittance is significantly higher for the frequency range investigated in the simulations. This agrees well with that of the experimental data and is also reported to be a general problem for plate girder bridges (Ito et al., 1991). The original Tacoma Narrows bridge (Farquharson, 1952), the Long Creek bridge (Ito et al., 1991) and the Kessock bridge (Owen et al., 1996), all with geometries resembling that of the Busan–Geoje bridge, suffered from extensive pitching motion. The same increased aerodynamic admittance is found for both lifting and pitching moment of the Øresund bridge and the Stonecutters bridge whose cross sections consist of multiple structures. The increased aerodynamic admittance may be explained by a aerodynamic interaction between different parts of the structure. Evidently the vortex shedding of one part of the structure can influence the aerodynamic forcing of a downstream part which may result in an increased pitching moment as discussed earlier in Section 3.4.

### 3.6 The effect of a turbulent oncoming flow

It is noticed in the simulations that the turbulent fluctuations of the oncoming wind causes the thickness and separation of the boundary layer on the bridge section to vary in time. This is mostly due to large fluctuations of the angle of attack caused by the turbulence in the oncoming flow. In Fig. 8 a flow image of a laminar simulation is compared to flow images of two selected time instances of a turbulent simulation which illustrates the instantaneous change in the boundary layer. As mentioned in Section 1 extreme fluctuations of the angle of attack may be experienced even at low turbulence intensities due to an instantaneous synchronisation of small flow scales. Such fluctuations may cause the flow to separate on the leading edge bridge section which results in a large change in the aerodynamic forces acting on the bridge section. The phenomenon of extreme angles of attack, though statistically well defined by means of the turbulence intensity, cannot be investigated by a spectral analysis but must be studied by the time signal itself. Examples of a time series where the aerodynamic force changes significantly is shown in Fig. 9 comparing the laminar simulation to the turbulent simulation for turbulence intensities of 2%, 5%, and 7%. For all time series it is seen that large fluctuations cause the lift force to deviate significantly from the mean value and affects the lift force for some time as the boundary layer of the bridge section recovers from the irregular vortex shedding.

![Fig. 6](image_url). The estimated aerodynamic admittance functions of the lift force $A_L$ (red/grey) compared to the aerodynamic admittance function of thin aerofoil theory $A_{L_0}$ (black), and available experimental result (dashed black/blue). The estimated Strouhal frequency is indicated by a vertical line (dashed black). (a) Great Belt East bridge, (b) Øresund bridge, (c) Busan–Geoje bridge, and (d) Stonecutters bridge.
4. Conclusion

The discrete vortex method was found to be a good method to model the effect of an oncoming turbulent wind. The resulting velocity power spectra of the simulated 2D flow were found to agree well with the semi-empirical spectra of 3D atmospheric turbulence used in the presented stochastic method to create the turbulent flow. In spite of being 2D, the simulated flow showed to preserve most of the statistical characteristics of the generated turbulence with a slight loss of the anisotropic properties which is believed to arise when converting the generated turbulent velocity field into vortex particles.

The modelling of an oncoming turbulent flow in a two-dimensional discrete vortex method was applied to various bridge deck designs in order to extract the aerodynamic admittance function of bridge sections. The method is tested on four specific bridge sections representing the current state of the art bridge decks used in suspension and cable stayed bridges. The estimated aerodynamic admittance functions were compared to available experimental data as well as to Liepmann’s approximation to Sears function used in thin aerofoil theory. The spectra extracted from the simulations show a general good agreement with the experimental results, and a deviation from thin aerofoil theory which is well documented in the literature of bluff body aerodynamics. A slight deviation was observed in the lowest frequencies which may to be caused by different integral lengths in the turbulence fields of the simulation and experiment.

A short investigation was presented to illustrate the instantaneous effects of the oncoming turbulence. Here it was shown that the method can be used to perform temporal analysis to take into account the effect of statistically extreme fluctuations (gusts). Here it was shown that even at small turbulence intensities extreme fluctuations in the aerodynamic force occur creating a noticeable difference in the time history of the aerodynamic forces at different turbulence intensities.
Hence the discrete vortex method was found to provide a useful tool to estimate the aerodynamic admittance function as well as investigate other aerodynamic phenomena of bluff bodies under the influence of a turbulent oncoming flow. The method is able to simulate a turbulent oncoming flow with a relation of length scales similar to that of a full scale bridge deck in atmospheric turbulence. Thus the method provides plausible results in a frequency range that significantly exceeded that of the experimental investigations. The parameters of the simulation are easily adjusted such that many different flow conditions can be investigated without extensive modifications, making the method a valuable design tool in bridge aerodynamics.

Acknowledgements

The authors wish to acknowledge support from the COWI foundation, the Danish Research Council (Grant no. 274-08-0258), and computer resources from the Department of Physics at DTU through the Danish Center for Scientific Computing (DCSC).

Appendix A. The modified von Kármán spectra of atmospheric turbulence

The spectral power density function $S_u$ and $S_v$ for the two velocity components $u$ and $v$ are proposed by the Engineering Science Data Unit (ESDU, 1993) as a modified von Kármán spectra

$$\frac{kS_u}{\sigma_u^2} = \beta_1 \left(1 + \frac{2n_u}{\beta_3} \right)^{2/3} + \beta_2 \left(1 + \frac{n_u}{\beta_3} \right)^{2/3}$$

$$\frac{kS_v}{\sigma_v^2} = \beta_1 \left(1 + \frac{8}{3} \left(4\pi \frac{n_v}{\beta_3} \right)^{2/3} \right)^{2/3} + \beta_2 \left(1 + \frac{2n_v}{\beta_3} \right)^{2/3}$$

with the reduced frequencies

$$n_u = \frac{kL_u}{U} \quad \text{and} \quad n_v = \frac{kL_v}{U}$$

and $F_1$ and $F_2$ being

$$F_1 = 1 + 0.455 \exp(-0.76 \left(\frac{n_u}{\beta_3}\right)^{-0.8})$$

$$F_2 = 1 + 2.88 \exp(-0.218 \left(\frac{n_v}{\beta_3}\right)^{-0.9})$$

$\beta_1 = 0.80$, $\beta_2 = 0.2$, $\beta_3 = 0.662$ are spectral coefficients specified in Harris (1990) and the integral length scales $L_u$ and $L_v$ are calculated by

$$F_1 = 1 + 0.455 \exp(-0.76 \left(\frac{n_u}{\beta_3}\right)^{-0.8})$$

$$F_2 = 1 + 2.88 \exp(-0.218 \left(\frac{n_v}{\beta_3}\right)^{-0.9})$$

$\beta_1 = 0.80$, $\beta_2 = 0.2$, $\beta_3 = 0.662$ are spectral coefficients specified in Harris (1990) and the integral length scales $L_u$ and $L_v$ are calculated by
\[
L_u = \frac{A_{u}^{3/2} \left( \frac{\sigma_{u}}{u_{*}} \right)^{3} y_0}{2.5 K_z^{1/2} \left( 1 - \frac{y_0}{h} \right)^2 \left( 1 + 5.75 \frac{y_0}{h} \right)}
\]

\[
L_r = L_u \left( 0.5 \left( \frac{\sigma_{u}}{u_{*}} \right)^{3} \right).
\]

(A.5)

\(A_{u}^{3/2}\) and \(K_z^{1/2}\) are parameters specified in ESDU (1993), \(u_\ast\) the shear velocity, \(h\) the thickness of the boundary layer and \(y_0\) the height above ground.

References


